Maria Renold

Intervals, Scales, Tones and the Concert Pitch c = 128 Hz

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TEMPLE LODGE

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Foreword to the First German Edition

From antiquity, repeated attempts have been made to reshape tonal laws and harmonic relationships, not merely for everyday music making but because people wished to find and bring order to human destinies in the tones. The right tones needed to be in harmony with the whole cosmos, the planets and the zodiac.

Today we hear and feel things differently. Much endeavour is going into bringing this to full conscious awareness, first of all aurally, which must be regarded as the only most genuine gauge, and then in making calculations to get a precise mathematical basis for what is heard.

Working for decades to test, clarify and bring light into these matters, the author has grown skilful in making fine distinctions. She has tested countless listeners and never ceased in her efforts to take Rudolf Steiner's suggestions seriously, interpret them and bring them into harmony.

She has established that there is a 'right' concert pitch which is a little below the concert pitch of modern pianos. It is c = 128 Hz. Differentiating between 'Dionysian intervals' and 'Apollonian Pythagorean true fifths' creates a scale with therapeutic qualities that we can experience. Subjectively speaking, c = 128 Hz has a kind of enveloping warmth around it.

Objectively Maria Renold set herself the difficult task of relating two statements made by Rudolf Steiner with one another. A lecture Rudolf Steiner gave in 1923 (in *Supersensible Man*) provides a golden thread for connecting them with the world. Our fifth post-Atlantean civilization can be understood if we characterize it with the aid of two phenomena—the fact that Michael, Archangel of the sun, has been the leader from 1879, and the power of Mars. These two spiritual streams come together in c = 128 Hz.

May the work of Maria Renold bring this alive for our understanding.

Karl von Baltz
December 1984

Translator's Preface to the English Edition

Because the translator was fortunate enough to be able to work closely with the author, this English edition is more of a reworking of the original than a direct translation. So, for example, even though the content is the same, chapter 8 was practically written anew. Many of the more specialized musical terms such as 'true' and 'just' were given by the author and the preference for the word 'tone' for that which sounds and 'note' for its printed representation is also due to this collaboration. The reference numbers throughout are bibliographic references and are therefore not in sequence. The Glossary and Appendix 2, which shows the result of Paul Davis's research into the twelve fifth tones tuning, are new. For an explanation of the pitch indications, see Glossary 'Pitch of a tone'. If further discoveries are made or questions arise, please contact the translator at bevis@bluewin.ch.

A big word of thanks goes to all who have given help and support in what has been a labour of love, but I wish to give a special word of thanks to Anna Meuss who turned my rather inept translation into acceptable English.

Bevis Stevens
Spring 2003

Preface to the Second German Edition

The new questions and answers that have come up since the first edition of this book appeared in 1985 have deepened and broadened the subject. Above all it has been possible to develop a new method of tuning the scale of twelve fifths for instruments with set tuning (e.g. the harp, bells, piano, etc.), and this is fully considered in the new Part Four, together with some closely related phenomena such as open fourths and fifths, the minimally increased octave and difference tones, the objective existence of which continues to be in dispute. Special attention has also been paid to 'absolute pitch', since this is often considered to make it difficult to become accustomed to a tuning at a lower pitch. The 'free tone' and a few suggestions to help one inwardly experience the 'quality' of pitches provide the subject matter for further chapters.

Agreement, and also criticism, in so far as they arise from true recognition and understanding of the subject, can only be a help in the work done in this field. I am therefore most grateful to everyone who has thus been contributing to the research, which is as yet far from complete.

A special word of thanks goes to my dear husband Adolph Renold for his consistent active support.

Maria Renold
Autumn 1991

Preface to the Third German Edition

Circumstances that are almost beyond comprehension caused a number of errors to appear in the second edition which distorted the meaning. Unfortunately it was too late when we discovered them. For this new edition the text has again been thoroughly reviewed. New research findings, also by others, have been added. This third edition also includes the colour charts* which were the first written records made in 1963 of these researches.

Maria Renold
Easter 1998

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* Not included in the English edition.

Introduction and General Outline

Music, as we know it, is perceived, sensed and experienced in both inner and outer hearing.[1] All consideration, investigation and judgement of musical phenomena must therefore begin with observations based on human hearing and inner experience. The importance of these words is such that they appear here at the beginning of this work where they cannot be overlooked, because even a fleeting glance through these pages reveals a lot of figures and one might rightly ask what all these abstract figures have to do with the human experience of music.

Intervals and tones can be experienced through inner hearing, through singing and by producing them on a musical instrument. If we sing, play a violin, blow a flute or tune a piano, we can hear and experience the note and interval relationships described in this book. For more detailed study, they can be reproduced on a monochord.* This book therefore also contains instructions on how to build a monochord, whose three strings enable the study not only of sounds but also of all intervals and triads. The interval and string length ratios can be measured on the monochord strings and used to describe the aural phenomenon. From this it is a small step to numerical representation of the intervals, a step originally taken by Pythagoras.[4,5] The monochord may thus be used to trace musical experience to its mathematical roots, which makes it much easier to present. Mathematics has the advantage over the spoken word of being more exact and unmistakable. Where the discussion is later in mainly mathematical terms, for example relating to the structural laws and form principles of a scale, it will be important to remember that the figures are not the primary element in observation. It is what is *heard* that is most important.

Reference must also be made to an important aural limitation. It is very simple today to reproduce an interval or note by means of physical apparatus, as is customary in well-equipped major recording studios. Frequencies and wave characteristics can be reproduced with tremendous accuracy, but the great disadvantage is that by being electronically produced they lose their qualitative content completely. They are at an exactly determined and determinable pitch, which can be successfully used to calibrate tuning forks, for example, but their individual quality is lost. In its place, a consistent, erosive quality arises in the structure of all notes and intervals produced, which must be due to the electricity.[6] Our sense of hearing therefore forbids the use of notes and intervals reproduced by electrical apparatus[7] [See chapters 15, 20 and 23.]

Great emphasis is generally put on the authenticity of musical phenomena. 'Genuine' and 'false' may be unfortunate terms, but practical experience shows that we are able to distinguish spontaneously between intervals which we call genuine or false. It will only prove more difficult if we have partially lost our musical sensitivity by accustoming ourselves to equal-tempered tuning or electronically distorted music. In present music literature only beatless intervals, the 'perfect consonances' also referred to by Pythagoras—the octave, fifth and fourth as well as the major and minor thirds and sixths—are called 'genuine'. They are not often mentioned, but experience shows that these few intervals are merely special cases among all the intervals that the ear feels to be genuine. Auletic intervals,[8,9] Pythagorean intervals (called true intervals throughout the rest of the work) and the 'formed' intervals described later in this book, as well as three, and only three, intervals of the equal-tempered scale—the equal-tempered tritone, minor thirds and major sixths—are also experienced as genuine. Although some of the latter do beat a little, they sound genuine to the ear, i.e., they sound 'right'. When the term 'genuine' is used in this work, it should therefore be understood in the wider sense given here.

In the first part of this work, intervals are discussed and the scales in which they appear are found. In order to understand and survey them more easily in their abundance, the new concept 'form principles

* Monochord. An ancient apparatus consisting of a resonance box with originally one and later several strings.[19,62]

of scales' is introduced. The form principles characterize the structural laws of a scale and are based on the three means that have been decisive in music from antiquity[5]—the arithmetic, geometric and harmonic means. The form principles must therefore essentially be described in mathematical terms. The form principles will also be applied in an attempt to provide a fuller explanation of Plato's famous account of how the world's soul was created.

The scale of twelve fifths will also be considered. This scale answers a real musical need as it allows non-tempered tuning of instruments with set tuning (piano, lyre, organ, etc.). The beauty as well as practicability of this new scale has already been attested by several hundred people with and without musical background. Part One thus establishes a basis for the new insights discussed in the book.

The second part of this work is devoted to the discussion of musical sounds, their nature and individual pitch. When we talk about singing or playing 'in tune' today, it is commonly accepted that one needs to play in such a way that the intervals are pure or correct. The individual pitch of a tone is however considered to be unimportant, as is evident from the variations in the concert pitch a^1. In London in the nineteenth century it varied between 427.7 and 455.1 Hz, in Paris between 373.1 and 563.1 Hz, until at the end of that century it was internationally set at 435 Hz. At the beginning of the twentieth century it went up to 440 Hz and since the Second World War to 448 and even 460 Hz for several of the bigger European orchestras. In contrast to this, hearing experiments with over 2000 people have shown that a demonstrable feeling for the quality of the absolute pitch of a note exists even among ordinary people. This is important, as it is known that the importance of a note's pitch was widely known in ancient Greece.[16] The pitch of a tone was considered to have a specific moral quality.

A fairly comprehensive chapter is devoted to $c = 128$ Hz—the old 'philosopher's C'. Taking the many oral indications given by Rudolf Steiner and the author's own far-reaching extended listening experiments, an attempt is made to show that $c = 128$ Hz is the only really suitable concert pitch for present-day Western people.

In the third part, the insights gained so far are used to discuss a number of suggestions made by Rudolf Steiner. Two were given to Kathleen Schlesinger,[8] two to the musician Wilhelm Lewerenz[17] and two for eurythmists—the 'tone-spiral',[18,65] and 'C is always prime'. It will be found that a human and artistic development of music is definitely possible. This entails tuning the Dionysian and auletic 22/22 Sun mode and the six other planetary modes to the generating note $c^5 = 4096$ Hz and the use of $c = 128$ Hz as concert pitch and common prime for all just major and minor scales in the circle of fifths, the equal-tempered scale, the true C major scale and the new scale of twelve fifths.

To enable readers to test the subject matter for themselves and if desired carry out their own sound and interval experiments, a tuning fork will be found on the cover of the book, and instructions on how to build a monochord suitable for the work at hand are given in Appendix 1.

Part One

INTERVALS AND SCALES

1 Hearing Observations

The following observations are not coincidental but are the fruits of years of musical activity. They were made during concert tours with the Busch Chamber Orchestra, by singing in various large and small choirs, making music with lay people and teaching adults and children.

Especially amongst violin and viola players, a distinct difference is apparent in the intonation of thirds and sixths, the 'imperfect' consonances. For example, some players play a calm and beatless major third, while others intone it somewhat larger, causing it to beat a little—though still sounding harmonically correct—and giving it a radiance or shining quality which the beatless third does not have. Similar observations can be made with minor thirds and major and minor sixths.[15]

The differences in intonation can be heard when wind or string players play thirds or sixths alone or together. Wind players mostly play the major third in the first size described (which sounds calm and beatless), while string players use both sizes of the major third, depending on which tones make up the interval concerned. It is important to note that the difference in intoning both types of major third thus observed remained constant and the same and was not affected by the origin or virtuosity of the musician.[15]

The calm, beatless and somewhat smaller major third is called the 'natural' or 'just' major third; the somewhat bigger, slightly beating and shining major third is today still called the 'Pythagorean'[13, 14, 15] or 'true' major third. Because the difference in the use of the two thirds cannot be put down to technical ability, it must originate in an intuitive difference in hearing (the same applies to the minor third and the major and minor sixths). This difference in hearing the imperfect consonances can lead to bitter arguments over intonation amongst concert players, the one usually blaming the inadequacy of the other. These conflicts cannot be reconciled with the help of present-day musicology and both types of intonation are so commonly met with in practice that it is not right to say that either one of them is 'correct'. Factual, unbiased consideration must be given to both possibilities. In the following chapters the true and just intervals as well as the two types of dissonant intervals (minor second and major second, tritone and major and minor sevenths) are therefore treated as equally valid musically. The following examples taken from music practice and literature may briefly illustrate this.

In the last movement, Allegretto con variazioni, of Beethoven's *String Quartet*, Opus 74, at the end of the first phrase in the Allegretto and the fourth variation, both violins and the cello play the G in octaves, while the viola plays $_1$B in the Allegretto, and b in the fourth variation, i.e., the major third. The Busch String Quartet, world famous at the beginning of the twentieth century, played this work frequently in the course of their illustrious career. After one of the first performances Busch said to the viola player: 'You played the B too low today.' A short time later, when they played the quartet again, the same thing happened and over the years this B became a real bone of contention between the two men; Busch always wanted to hear the B played higher than the viola player played it. After more than 20 years of playing concerts together, Busch lost patience. After a concert he shouted at his violist: 'Why did you play that B so low again today?' The equally heated reply was: 'Because I hear it that way.' Both men had 'absolute pitch' and both were among the greatest concert artists of their day. It therefore could only have been an objective difference in hearing the major third.

An opposite example. An orchestra conductor who was also a wind player complained that the strings always played the major third in cadences too sharp for his ear. He wished it to be quiet and beatless.

A third example. Tape recordings of the renowned virtuoso David Oistrakh showed that he usually played in true intonation and only for long notes or when in unison with the piano did he react with lightning speed and conform to the tempered tuning of the piano.[11b]

Hermann Helmholtz had already discovered and described this difference in intonation of the major third in his day.[15]

This is easier to hear on a violin. Tune all four strings exactly to perfect (beatless) fifths: e^2, a^1, d^1, g. Then play b^1 with the first finger on the A string as a perfect fourth to the open E string. At the same time, place the third finger on the D string as perfect octave to the open G string. Now play g^1 and b^1 together as a double stop. This produces the major third which Busch wished to hear. It beats slightly but has a wonderful light, a shining strength. It is a 'true' major third. Bow or pluck it several times in order to be able to remember it (if it sounds harsh it has been played too large). Now play the b^2 as a harmonic by placing the second finger very lightly on the G string at a point where b^2 sounds two octaves and a major third higher than the open G string. The previously played b^1 on the A string is now played correspondingly lower to sound as the lower octave to the b^2 harmonic. Now play this b^1 together with g^1 (higher octave to the open G string). Now a smaller major third sounds. If it is pure it will sound without beating, clear and peaceful and the two octaves lower difference tone G will be heard sounding with it. When this b^1 is played together with an open E string, the resulting fourth will be too big and will sound impure and intolerable to the ear. This is the third which the violist played and the conductor had wished for. It is the 5:4 major third,[10,15] which, being beatless is also called 'just'.

Such examples taken from concert practice could be multiplied a hundred or even thousandfold. They point to a variance in hearing and experiencing the imperfect consonances with which concert musicians wrestle on a daily basis.

It may be said that Western people can be divided into two groups according to how they hear and experience imperfect consonances and dissonances. The one group recognizes only just intervals, and, if known, their mirror-images, which are the intervals of the early Greek aulos modes.[8,9] People in the second group also love true intervals and consider them a valid means of musical expression.

It is of historical interest that these two groups of people have their roots in ancient Greece. Legend tells of a competition between the lyre player Apollo and the flute player Marsias. The decision went in favour of Apollo. The flute of Marsias was tuned to just intervals and goes back to Pan and Dionysus. As a result, this musical direction is also called the Dionysian stream. The lyre of Apollo,[3,5] tuned to fifths, gave rise with its true intervals to the Apollonian stream of music. Both terms, Apollonian and Dionysian, may be used in this sense today.[7,72,73]

To avoid possible misunderstandings, it has to be stressed that both the perfect consonant intervals and the 9:8 major second belong to both streams. Octaves, fifths, fourths and major seconds are, so to speak, 'naturally' contained within the 'just' tone rows of the Dionysian stream and belong for example to the basic structure of aulos Venus mode 24/24 and the altered aulos Mars mode 20/20. In the Apollonian stream the perfect consonances and the 9:8 major second are formed by dividing the twelfth and its octaves by the arithmetic and harmonic means (see chapters 3–12). The difference between the two streams is thus only to be found in the imperfect (= third, sixth) and dissonant (= second, seventh, diminished and augmented) intervals. This must be added because it is often mistakenly maintained that our 'just' scales with their just thirds, sixths, sevenths and true-tone dominants and subdominants are a fusion of both streams.

Further observations, dealt with at length in a later chapter, were made playing a freely tuneable instrument with an equal-tempered instrument of fixed tuning—violin or oboe with piano, for example. As the freely intoning string or wind player can and must defer to the tuning of the piano, this means that he or she must play intervals that sound unmistakably untrue and even false to the ear. Though this fact is uncomfortably noticeable and relatively common with fourths and fifths, its importance is not sufficiently acknowledged. It will be considered in chapters 4, 9 and 21 of this book. There it will be explained why three of the mostly false intervals of equal-tempered tuning sound true to the ear, these being the equal-tempered tritones, minor thirds and major sixths.[11b]

If one wants not only to know but also to understand the rich variety of the above interval phenomena, it will be necessary to study the just-tone row (chapter 3), the true-tone row (chapter 4) and the structures of scales (chapters 6–13). This will be attempted in the chapters that follow. For reasons already given, such a study does involve the extensive use of numbers.

2 Terminology Used for the Different Types of Intervals, Tones and Scales*

Before we go further, a clarification of the terms used for the different types of intervals, tones and scales needs to be given. For this we will have to anticipate some of the things that will be considered in later chapters.

From what has been said in chapter 1, it follows that five different types of intervals and tones will be discussed:

1) those found in the commonly used 'just' major and minor scales;
2) those in the early Greek aulos modes;
3) those which make up the 'true' scales (i.e. the church modes);
4) those appearing for the first time in the scale of twelve fifths;
5) those belonging to the equal-tempered scale. The ear tends to experience these as 'falsified', however, and therefore they are only discussed once.

It is reasonably well known that the first three types of intervals and tones are contained in the overtone and undertone rows when these are extended.† In theory they go on to infinity, of course.

To make the subject matter of this book comprehensible it is imperative that the differences are clearly defined and named. The following three categories are therefore applied, listing first the intervals, then tones of the overtone and undertone rows, followed by the scales in which these tones appear.

Just

a) **Just intervals:** all intervals that appear in the 'just' major and minor scales and are bordered at *one* end by a tone which, as far as is known, does not belong to the true-tone row of C.‡
These intervals include

the just minor second 16:15
the small just major second 10:9
the beatless just minor third 6:5
the beatless just major third 5:4
the beatless just minor sixth 8:5
the beatless just major sixth 5:3
the beatless just minor seventh 9:5
the beatless just major seventh 15:8

b) **Just tones:** all tones creating the above intervals that are not contained in the true-tone row of C, e.g. just E (fifth overtone of C) and just B (fifteenth overtone of C), etc.

c) **Just scales:** all major and minor scales whose scale degrees III, VI and VII are just tones when they begin on true tones and true tones when they begin on just tones.

* See also Glossary.

† See chapter 3.

‡ Regarding the incongruity between just and true intervals, refer to Tables 22 and 26 in chapter 21.

True

a) **True intervals (also known as Pythagorean or twelfth-tone and fifth-tone intervals):** this description refers exclusively to intervals where both tones belong to the true-tone row.

b) **True tones:** all tones belonging to the true-tone row of C that make up true intervals.

c) **True scales:** scales consisting exclusively of true tones from the true-tone row of C.

Modal

a) **Modal intervals:** intervals arising in the undertone-row with the proportions 8:7, 11:10, 12:11, 13:12, 14:13 and 15:14 which do not appear in either major or minor scales, but are intervals belonging to the early Greek aulos modes.

b) **Modal tones:** the seventh, eleventh and thirteenth undertones.

c) **Modal scales:** aulos modes.

Thus, taking the tone E as an example: just E is the fifth overtone of C; just E and C make up the just major third 5:4. True E is the E of the true tone row of C; true E and C make up the true major third 81:64. Modal E is the thirteenth undertone of C, which corresponds to the third step of the Hypodorian aulos mode and makes the modal major third 13:16.

These three examples may serve to clarify the names of the different types of intervals, scales and tones. The given definitions clearly identify all the various types of intervals, tones and scales and make it easier to follow the often complicated subject matter. The complexity is deemed necessary for a healthy further development of music.

The scale of twelve fifths

a) The geometric mean tones of the scale of twelve fifths hardly need a special nomenclature. Their names Gelis, Delis, Alis, Elis and Belis point to their newness with regard to the tonal material used so far. The names were chosen to express the fact that they are neither sharps nor flats,* but geometric mean tones.

b) The intervals which these five tones create together with the other seven non-altered true tones are called 'formed intervals'. They all sound genuine to the ear and, with the exception of the equal-tempered tritone, seem to occur here for the first time.

The tuning based on the scale of twelve fifths is here called the twelve fifth-tones tuning.

To give readers an overview of all the intervals dealt with in Part One, Table 14 at the end of chapter 13 gives the interval sizes in cents to allow comparison of their size. It also shows that all intervals (perfect consonances,† imperfect consonances and dissonances), whilst they may bear the same names, are different in size, depending on whether they are just, true, aulos, equal-tempered or formed intervals.

* NB in the German language Ges means G flat and Gis means G sharp. Gelis contains both names. Refer to chapters 6, 12, 13 and Table 32 for geometric mean tones. (Translator's note.)

† Even octave 2:1 sounds untrue to the ear and is therefore usually played a bit larger. In the scale of twelve fifths it is also a bit larger (*c.* 3 cents) than 2:1. See chapter 22. Regarding the fifths and fourths, see chapters 13 and especially 22 and 23. (Translator's note.)

3 Overtone and Undertone Rows. Just Intervals

The overtone row*

The tones of the overtone row (called partials) ascend from the fundamental in regular order, resulting in a series of intervals and are natural phenomena up to the sixteenth overtone.† Successive partials increase by a constant amount in relation to the frequency of the fundamental tone. In other words, the ranking number of the partial, multiplied by the frequency of the fundamental, gives the frequency of the partial in question.

If, in order to keep the overtones within hearing range for as long as possible, we take c as the fundamental and let it resonate unhindered on the A string, then in addition to c the following tones will also be heard:

Overtone row

Abbreviations: j. = just, m. = modal

This row is a part of Table 1 and shows how the naturally arising row of tones ascends.‡ Every two consecutive overtones give the intervals shown in Table 1. The ranking numbers of the partials give the numerical proportions of the intervals. Therefore 2:1 = octave, 3:2 = fifth and 4:3 = fourth. These first three intervals are called 'perfect' because they were previously considered to have only one consonant sounding size.§ As soon as any of these intervals are enlarged by a minor second, they become dissonant augmented or diminished intervals. Then follow the major and minor thirds 5:4 and 6:5. The third has two forms and is therefore called an imperfect consonance. The difference in size between the two thirds is a minor second. Both are beatless, and may therefore be called pure (so long as this term is understood to mean beatless and not 'perfect' as with perfect consonances). All further consecutive intervals in this row are dissonant seconds.

It is interesting that none of the sixths, sevenths or tritones occur between two adjacent partials. They are produced in a different way. The just major sixth arises between partials 5:3 (missing out the fourth

* The partials of both the overtone and undertone rows are numbered from the first tone = No. 1 and not, as is sometimes done, from the second tone = 1.

† The German language bears witness to this fact in that it calls the overtone row *Naturton-Reihe*, i.e., 'natural tone row'. (Translator's note.)

‡ The infinitude of the row, which is easily realized by calculation, disappears from the realm of perception because of the limitations of the human ear. In reality, the row begins in the unhearable region below $_2C$ = 16 Hz and continues into an unhearable area above 40,000 Hz. Tones that lie outside the limits of human hearing can be brought into the audible range by 'octave displacement'.

§ See second footnote on p. 8.

Table 1
Overtone and undertone rows and their intervals on c = 128 Hz

Tone	Partial	Hz	Ranking-number Intervals: Cents	Ranking-number Intervals: Name and proportion	Intervals with leaps	Cents to C
*c^4	16	2048.000	111.731	j. semitone 16:15	j. ma. seventh 15:8	1088.269
b^3	15	1920.000	119.443			
flat b flat^3	14	1792.000	128.298			
sharp a flat^3	13	1664.000	138.573			
*g^3	12	1536.000	150.637			
fau^3	11	1368.000	165.004			
e^3	10	1280.000	182.404	small j. whole tone 10:9	j. mi. seventh 9:5	1017.597
*d^3	9	1152.000	203.910	whole tone 9:8	mi. seventh 16:9	996.090
*c^3	8	1024.000	231.174	aug. whole tone 8:7	small j. mi. seventh 7:4	968.826
flat b flat^2	7	896.000	266.871			
*g^2	6	768.000	315.641	j. mi. third 6:5	j. ma. sixth 5:3	884.359
e^2	5	640.000	386.314	j. ma. third 5:4	j. mi. sixth 8:5	813.686
*c^2	4	512.000	498.045	fourth 4:3		
*g^1	3	384.000	701.955	fifth 3:2		
*c^1	2	256.000	1200.000	octave 2:1		
*c	1	128.000		FUNDAMENTAL		
*C	1/2	64.000	1200.000	octave 1:2		
*$_1$F	1/3	42.667	701.955	fifth 2:3		
*$_1$C	1/4	32.000	498.045	fourth 3:4		
$_2$A flat	1/5	25.600	386.314	j. ma. third 4:5		
$_2$F	1/6	21.333	315.641	j. mi. third 5:6		
sharp $_2$D	1/7	18.285	266.871			
$_2$C	1/8	16.000	231.174	aug. whole tone 7:8		
$_3$B flat	1/9	14.222	203.910	whole tone 8:9		
$_3$A flat	1/10	12.800	182.404	small j. whole tone 9:10		
$_3$Geo	1/11	11.636	165.004			
$_3$F	1/12	10.667	150.637	$\frac{3}{4}$ tone 11:12	→Hypodorian aulos mode 16:16	
flat $_3$E	1/13	9.846	138.573			
sharp $_3$D	1/14	9.143	128.298			
$_3$D flat	1/15	8.533	119.443			
*$_3$C	1/16	8.000	111.731	j. semitone 15:16		

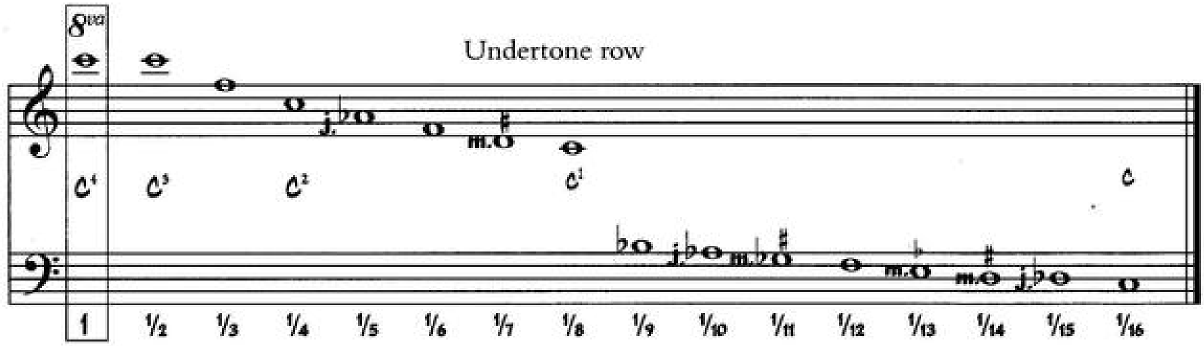
* True-tone rows. *Abbreviations:* aug. = augmented, dim. = diminished, ma. = major, mi. = minor, t. = true, j. = just.

partial) and the just minor sixth is found between partials 8:5 (leaving out partials 6 and 7). Likewise the just minor seventh arises between the partials 9:5, the just major seventh between 15:8, the 'very small seventh', proportionally lying between major sixth and minor seventh, has the ratio 7:4, while 45:32 gives the just augmented fourth (i.e., just F sharp:C) and 64:45 gives the just diminished fifth (i.e., C:just F sharp).

For the purpose of our study, the section between the eighth and sixteenth partials is of particular importance. This is the area between c^3 and c^4 in Table 1. The eight intervals produced by these partials are the following: 9:8, 10:9, 11:10, 12:11, 13:12, 14:13, 15:14, 16:15. This sequence of intervals shows correspondence to those of the just major scale but is not identical with it (see chapters 7 and 9).

The undertone row

Unlike the overtone row, the undertone row of a fundamental is not a natural phenomenon. It must be artificially produced. It has however been known and made use of in music from antiquity. It is more difficult to demonstrate. One does not use the whole length 1 of the monochord string as the fundamental; instead, 1 is divided into equal lengths and the smallest length (with the highest sound) becomes the new unit 'u'. When first u, then 2u, 3u . . . and ultimately the full length of the string are made to sound, the descending interval row octave, fifth, fourth, major third etc., becomes audible, the exact mirror image of the intervals of the overtone row.*



Abbreviations: j. = just, m. = modal

For Table 1, the undertone row has been transposed down to the fundamental of the overtone row, c, so that it can easily be compared with the overtone row. The partials of the undertone row need to be numbered in fractions. To find the frequency of a tone, one then multiplies the frequency of the fundamental (in this case c = 128 Hz) with the relevant fraction. This produces the frequency of the tones and the interval series of the undertone row as given in Table 1.

The intervals of the undertone row between $_2C$ and $_3C$ are especially important as they have a correspondence with the Hypodorian aulos mode.[8,9] This will be dealt with separately in chapters 8 and 9.

Summing up, therefore, examination of the intervals of the overtone and undertone rows of c, as they are presented in Table 1, shows that they are mirror images and, as is shown by the cents, the intervals of both rows get smaller in the same way. Between the first and sixteenth partial, consequently within the area of the first four octaves, lie all the diatonic intervals of the just scales. They are characterized by simple numerical proportions. The chromatic just intervals, e.g. the augmented fourth 45:32 or the diminished fifth 64:45, are first found in the octave between the 32nd and 64th, or 1/36 and 1/64, partials, the imperfect consonances and the true dissonances in the octaves that follow immediately between 64 and 256, or 1/64 and 1/256, and the true augmented and diminished intervals in the octave between 512 and 1024, or 1/512 and 1/1024. Not all of these octaves are shown in Table 1 as this would make it extremely complex. The ever-decreasing interval size becomes unsatisfactory to the ear as well and they are hardly made use of in music. As a rule, the overtone and undertone rows are not taken to these extremes. However, extending them like this simplifies the comparison between the beatless imperfect just intervals and the true intervals of the same name. The fifth tones which determine the true intervals follow their own structural principle.[1,2,4,5]

* The limited length of the monochord makes the demonstrable row finite, though mathematically it is infinite.

Although they are partially contained in the overtone and undertone rows, the true tones need to be treated as a row of their own. They are therefore considered in the next chapter where an initial comparison between true and just intervals will also be made.

4 The Twelfth-tone Row, the True Intervals and a Comparison with the Just Intervals

The true tones that make up the twelfth-tone row are contained in the overtone and undertone rows that go on to infinity; however, the twelfth-tone row itself has a very different structure. The partials of the overtone and undertone rows relate to each other through their vibration frequencies in a linear progression of whole numbers (overtone row) or fractions (undertone row). As the ear does not hear the vibration ratios linear-additively but logarithmically, these rows are conceived as a progression of logarithmically diminishing interval sizes. In contrast to this, each successive true tone is an interval of a twelfth (i.e. an octave plus a fifth) away from the last—hence the name twelfth-tone row. This means that the true tones stand in a vibration ratio of 3:1, or 1/3:1, to each other. In other words, the frequencies of the tones in the row are based on powers of the number 3. They thus form a separate, clearly discernible row of constant twelfths within the overtone and undertone rows.*

It is interesting and also significant that the interval of a twelfth can be observed as a natural phenomenon on low-pitched string instruments. If a monochord, piano, cello or viola string is tuned to either c or C and is struck, plucked or bowed and left to resonate unimpeded, then, at the moment of sounding, the fundamental is heard and, as the tone dies away, the twelfth (not the octave or third etc.) is clearly discernible and resonates for a long time afterwards.†

If, in terms of the overtone and undertone rows, the fifth (3:2) is taken to be a reduction of the twelfth (3:1), then the twelfth can be considered to be a primary interval. We may even regard it as a type of primordial phenomenon. Due to purely phenomenological aural reasons, the twelfth must therefore be given a special place within the overtone and undertone rows. This is shown in Table 2.

The vibration ratios of the twelfth row are based on powers of the number 3, and therefore a characteristic of this row is that no tone can appear twice. The interval of an octave never occurs. This row can also be called the fifths row because each successive tone, if transposed within the same octave, forms a perfect fifth, or if the higher tone, through the transposition, ends up below the lower, its inversion the fourth. With the interval between any two successive twelfth-row tones always the same size, any of these tones can be used as a tone of departure. Taking C as the first tone and going 19 twelfths up and 15 twelfths down will give the 35 fifth tones known from ancient times:

F♭♭, C♭♭, G♭♭, D♭♭, A♭♭, E♭♭, B♭♭, F♭, C♭, G♭, D♭, A♭, E♭, B♭, F, C, G, D, A, E, B, F♯, C♯, G♯, D♯, A♯, E♯, B♯, F♯♯, C♯♯, G♯♯, D♯♯, A♯♯, E♯♯, B♯♯.

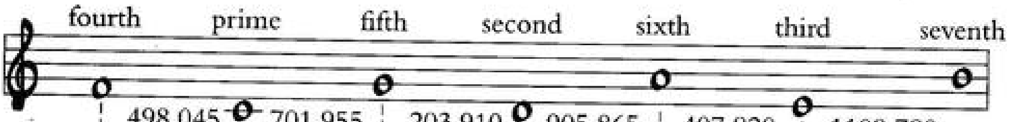
For the moment, let us take the seven non-altered tones of the 35 and place them within one octave. If C is kept as the first tone with the number 1, then the other six fifth tones form the true intervals shown in the diagram at the top of p. 14.

This extract of the twelfth row shows that any seven successive tones give the true interval degrees of a complete diatonic scale. Thus any two neighbouring tones create a fifth or its inversion a fourth, for example; the outer tones of any three neighbouring tones create a true major second and its inversion a true minor seventh, and the outer tones of any six neighbouring tones create the true majorseventh and the true minor second (limma).

If we wish to describe the interval proportions, the chosen beginning tone of the twelfth row may be indicated by the number 1, the ascending tones by 2, 3, 4 . . . and the descending tones by –2, –3,

* The importance of the twelfth-tone row for a true understanding of the just intervals will be considered in chapter 21.

† The phenomenon is difficult to observe on higher strings because the twelfth sounds much more softly. It cannot be observed on wind instruments, as the tone only lasts for as long as it is played and therefore neither resonates nor dies away.



Cents:	498.045	701.955	203.910	905.865	407.820	1109.780	
Overtone row Proportion:	4:3	3:2	9:8	27:16	81:64	243:128	
Fifth-tone row position number:	−2*	1	2	3	4	5	6

* Position number in the descending fifths-tone row from C.

Table 2
The twelfth-row tone proportions of the diatonic intervals

Tone-number of the twelfth-tone row		Set within the octave:		Comparison
Tone number proportion	Tone example	a) Interval b) Complementary interval to the octave	Cents	Overtone-row proportion
2 to 1	g^1 : c	a) fifth	701.955	3:2
*2 to 1	$_1$F : c	b) fourth	498.045	4:3
3 to 1	d^3 : c	a) major second	203.91	9:8
*3 to 1	$_3$B flat : c	b) minor seventh	996.09	16:9
4 to 1	a^4 : c	a) major sixth	905.865	27:16
*4 to 1	$_4$E flat : c	b) minor third	294.135	32:27
5 to 1	e^6 : c	a) major third	407.82	81:74
*5 to 1	$_6$A flat : c	b) minor sixth	792.18	128:81
6 to 1	b^7 : c	a) major seventh	1109.775	243:127
*6 to 1	$_7$D sharp : c	b) minor second (limma)	90.225	256:243
6 to 1	f sharp^9 : c	a) augmented fourth	611.731	729:512
1 to *6	c : $_9$G flat	b) diminished fifth	588.269	1024:729
*2 to *8	$_1$F : $_{10}$C flat	b) augmented fourth	611.731	4374:3072 = 729:512
6 to *2	b^7 : $_1$F	a) diminished fifth	588.269	3072:2187 = 1024:729

* Position number in the descending fifths-tone row of C.

−4 . . . as was done above. This is the method used in Table 2, and the numbers show that the simplicity of the numerical ratios gained in this way for the true intervals is similar to that which we have already become used to for the just intervals. The true intervals thus gain in advantage for those who prefer straightforward numerical relations. Because the Hz between the members of the twelfth row are different from those between the partials of the overtone and undertone rows, another numerical representation of the intervals arises that is just as 'correct' for the true intervals as the known method is for the just intervals.

In chapter 1, attention was drawn to the fact that the imperfect true intervals and their corresponding intervals differ not only in size but also in quality, and how this can often give rise to bitter arguments about intonation between musicians. It therefore seems important to clearly illustrate which intervals in the two rows are different from each other, and the nature and extent of the difference.

According to Tables 1 and 2, intervals in both rows that are identical in quality and size are fourths, fifths, one of the major seconds, and one of the minor sevenths. Intervals that are of different size and therefore also of different quality are most widely used when playing music today. They are the major and minor thirds; major and minor sixths; major seconds and minor seconds; major sevenths; augmented fourths and diminished fifths (tritones). In the comparison given below, the numerical ratios of the overtone row are transposed through octave displacement into a higher octave where either a common numerator or denominator is found and the cents are also given.

just major third 5:4 = 80:64 or 386.314 cents	**true major third** 81:64 or 407.820 cents
just minor sixth 8:5 = 128:80 or 813.687 cents	**true minor sixth** 128:81 or 792.180 cents
just minor third 6:5 = 96:80 or 315.641 cents	**true minor third** 32:27 = 96:81 or 294.135 cents
just major sixth 5:3 = 80:48 or 884.359 cents	**true major sixth** 27:16 = 81:48 or 905.865 cents
just minor second 16:15 = 256:240 or 111.731 cents	**true minor second (limma)** 256:243 or 90.225 cents
just major seventh 15:8 = 240:128 or 1088.269 cents	**true major seventh** 243:128 or 1109.775 cents
just diminished fifth 64:45 = 1024:720 or 609.777 cents	**true diminished fifth** 1024:729 or 588.269 cents
just augmented fourth 45:32 = 720:512 or 590.223 cents	**true augmented fourth** 729:512 or 611.731 cents

The qualitative difference between the just and true major thirds is that the former sounds peaceful, bright and courageous, but may also be felt to have an underlying sharpness. The latter beats a little but sounds as though it radiates light. The difference between the two minor thirds can be experienced in that the just minor third sounds beatless and sad, but with an underlying voluptuousness; on the other hand, the true minor third sounds excited.*

A very unexpected and remarkable difference is found between the major sevenths. It is common knowledge that a just major seventh 15:8 with 1088.269 cents must be resolved into a neighbouring consonance, either upwards to the octave or downwards to the major sixth, if the ear is to be satisfied and the incredible tension released. But the noticeably bigger true seventh 243:128 of 1109.775 cents, although it obviously has a slightly dissonant character, sounds bright and peaceful, almost in suspension and exerts absolutely no compulsion on the human ear to be resolved. One can enjoy its brightness and beauty with equanimity and joy and let it resound unresolved.

One of the most interesting comparisons may be made between the two pairs of tritones—diminished fifths and augmented fourths. In the theory of harmony it is known that a diminished interval needs to be resolved inwards, and an augmented interval outwards. This rule arises from the desire that the human ear experiences upon the sounding of such dissonances, and with the correct resolution of the *true* tritones this desire is totally satisfied. However, if one plays a *just* diminished fifth that is perfectly in tune, for example, there is no wish to resolve it to a major third. Just the opposite—aurally one wants to enlarge it to a minor sixth. The opposite holds true for the *just* augmented fourth; if one truly follows one's ear, it wants to be resolved downwards into the major third.

These apparently contradictory aural experiences become understandable when the cents of both types of just tritones are compared with those of the true tritones and then read 'crossed-over'. One then

* For the cause of these phenomena, see chapter 21.

notices that the just augmented fourth of 590.224 cents is only 2 cents bigger than the true diminished fifth of 588.269 cents, and that both intervals lie below the middle of the octave of 600.000 cents. It is no wonder that, following the ear, we want to resolve both downwards. The ear only desires the tritone to be resolved upwards when it is bigger than the middle of the octave. Therefore the opposite is the case with the just diminished fifth of 609.777 cents. It is bigger than half the octave and only 2 cents smaller than the true augmented fourth of 611.731 cents. The ear therefore demands that they are resolved upwards to the minor sixth.

Augmented intervals have a rather over-tensed quality, while diminished intervals are experienced as rather cramped. Therefore, one may call the former luciferic in tendency and the latter ahrimanic.[26,27,28,29] In the case of the true tritones, their names (diminished and augmented) agree with the aurally perceived characteristics. In the case of the two just tritones, this correspondence is the opposite, with name and the aural experience contradicting one another. The actual aurally experienced quality of these two dissonances is obscured by the names they are given, so that the name does not tell us what we are dealing with. (With the help of the monochord, the described intervals can be easily heard and experienced.)

5 The Intervals of Equal-tempered Tuning

Up to this point, only intervals that sound genuine to the ear have been discussed. However, another tuning is the equal-tempered tuning to which nearly all pianos* are tuned. Even the name 'equal-tempered' indicates that the intervals are altered with this tuning and although most people do not know that the piano is deliberately tuned impurely they sense it. Why is equal-tempered tuning used?

Firstly, on a wind or string instrument, the musician produces the necessary tones and intervals as they are needed and has to intone each tone anew. On the piano, this is impossible. The strings have to be pre-tuned because the musician cannot retune whilst playing.

Secondly, the range of the piano is a bit more than 7, that of the harp 6½ and that of a big organ 9 octaves. Therefore, these instruments cannot be tuned to true fifths, because when one accumulates 12 perfect fifths, one upon the other, in order to get the 12 needed for minor seconds in the chromatic octave, the result is an interval that is 24 cents, or a Pythagorean comma (true minor second) greater than 7 octaves. (The octave of 1200 cents × 7 octaves = 8400.000 cents; the fifth of 701.955 cents × 12 = 8423.460 cents.) The impurity of the octave caused by this difference is so great that the human ear finds it intolerable.

Since keyboard instruments have come into use, different methods of tuning were attempted in order to make a free passage through all major and minor keys possible. 'Mean-tone' tuning, which was in general use over a longer period of time, worked well in certain keys but others sounded so impure that one could not make music with them. This was unsatisfactory, therefore. Finally, Andreas Werkmeister (1645–1706) divided the difference of the 7 octaves (*c.* 24 cents) into 12 equal parts, thus reducing the size of each fifth by 1/12 of the difference. He then put these tempered fifths once again within the octave. A tuning method was thus found which, even though it mainly consisted of 'false' intervals, gave a seemingly useful solution. This is the equal-tempered system of tuning, which seems to offer the desired freedom to play in all keys. Later it was discovered that most pianos sounded better if the middle region was tuned to perfect octaves and equal-tempered fifths, and both ends to slightly enlarged octaves and somewhat less tempered fifths. This is the method many piano tuners use today. Neither true nor just but merely the altered equal-tempered intervals are therefore heard today on normally tuned organs and pianos.

The inaugurators of equal-tempered tuning knew that they altered the intervals but held the view that the resulting impurity would be so small that it would be tolerable to the ear and could be accepted as part of the price for gaining free passage through all major and minor keys in the 'circle of fifths' without the troublesome addition of many extra keys and strings, etc. But even though equal-tempered tuning came to be widely used, their view has by no means been confirmed. Many instrumentalists and singers suffer from this falsification of the intervals. This is most distinctly experienced with the fifths and fourths, which simply sound false, but one cannot help feeling that there is also something wrong with the intonation of the major thirds.

On the piano, four or more voices often sound simultaneously, which means that several equal-tempered intervals are heard at one time. This may be the reason why, without exception, all equal-tempered intervals have been described as false. But if one aurally tests each individual interval, one finds that only eight of the twelve intervals actually sound wrong, while four are clearly experienced to be genuine. These are the octaves, the equal-tempered tritones, the equal-tempered minor thirds and the equal-tempered major sixths. It is very important to realize clearly that along with the octaves, the other

* The same applies to the harp, organ, lyre, tubular bells, etc.

three of these four intervals, which appear in neither true nor just intonation, are also experienced by the ear to be genuine.

Is the above aural experience subjective and random or does it follow generally understandable musical laws? How can we show why the ear experiences certain intervals to be genuine, whereas others, though acoustically and mathematically very similar, are justifiably rejected? To answer these questions, the structure of the scales in which the intervals concerned appear will be considered. The term 'form principles' will be applied to these structures and laws. The most perfect of all intervals,[4] the octave, which encloses all present-day intervals, will be our starting-point. The next four chapters will thus be concerned with the form principles of the different scales.

6 The Octave and the Harmonic, Geometric and Arithmetic Means in Music

The importance for music of the interval of the octave and the three mean formations—the harmonic, geometric and arithmetic means—is well known. The octave will be discussed first, and then the three mean formations. It will be shown how the octave gradually gained in importance in the course of musical development. We have seen that the octave is not to be found in the twelfth-tone row but is a natural phenomenon within the overtone and undertone rows. This may be the reason why the early Greek aulos modes were already contained within the octave, whereas all the old true-tone scales were originally pentatonic (e, d, b, a, g), then heptatonic and only much later reorganized into octatonic scales.[4,5] Prior to Pythagoras, there were seven possible pentatonic scales. However, in pre-Christian times, they progressed in a descending and not ascending direction as our present scales do. Such a scale was called a heptachord (7 tones = 6 steps of a true major second) and consisted of two connected tetrachords (4 tones = 3 steps of a major second), whereby the second tetrachord began on the tone on which the first ended:

A,	G,	F,	E									
			E,	D,	C,	B						
	G,	F,	E,	D								
				D,	C,	B,	A					
		F,	E,	D,	C							
					C,	B,	A,	G				
			E,	D,	C,	B						
						B,	A,	G,	F			
				D,	C,	B,	A,					
							A,	G,	F,	E		
					C,	B,	A,	G				
								G,	F,	E,	D	
						B,	A,	G,	F			
									F,	E,	D,	C

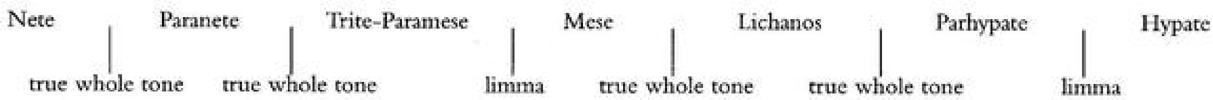
The true-tone tetrachord was arranged in four different ways, depending on where the minor second occurred or was absent. The Greeks called them tetrachords named after the tribes in which, as far as they knew, they originated.[4,5] These were

the **Dorian tetrachord** of:		
true major second 9:8	true major second 9:8	limma (true minor second) 256:243
the **Phrygian tetrachord** of:		
true major second 9:8	limma 256:243	true major second 9:8
the **Lydian tetrachord** of:		
limma 256:243	true major second 9:8	true major second 9:8
the **Mixolydian tetrachord** of		
true major second 9:8	true major second 9:8	true major second 9:8

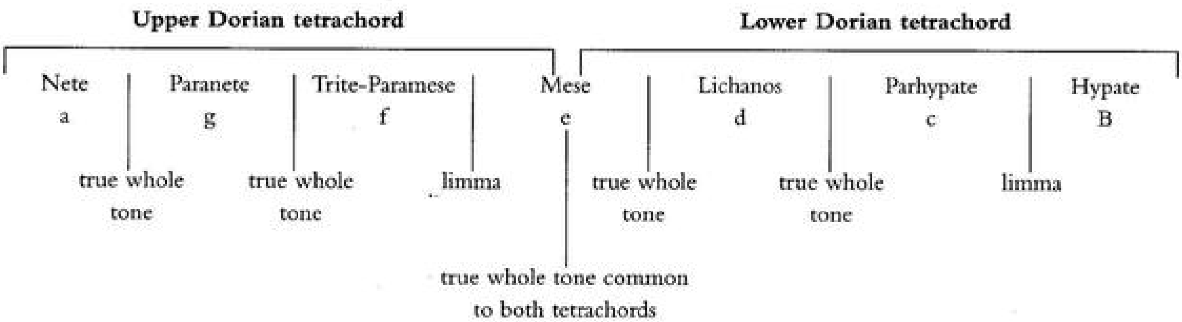
The Dorian tetrachord was brought into connection with the Sun,[4] and was considered to be the standard. A heptachord consisting of two connected Dorian tetrachords was therefore the principal or fundamental scale of the Apollonian true-tone stream[3,4] (see chapter 12).

The development of the Dorian heptachord is described as follows in Greek legend. Music came into existence on the four-stringed lyre which Hermes had built for Apollo, the god of the Sun. Apollo gave the lyre to his disciple Orpheus. Later Torrebus, son of Atys and king of the Lydians, added a fifth, and Hyagnis, king of the Phrygians, a sixth string. The lyre was then cast into the sea from whence Terpander rescued it. He is supposed to have given it a seventh string and reversed it with the help of Egyptian priests. In this form the lyre was then handed down to posterity.[5] This version, whereby the lyre of Apollo had four and not only three strings, tuned to the tones of the four elements, coincides with the indications given by Rudolf Steiner. The base string, E, was tuned to the tone of the earth, the G string to the tone of fire, the A string to the tone of the air, and the D string to the tone of water.[20]

Terpander is regarded as the founder of historical Greek music, and the Dorian true-tone heptachord therefore bears his name. He is supposed to have lived in the seventh century before Christ. Nicomachos of Gerasa reports that the heptachord of Terpander was a row of descending tones arranged in the following way:[4]



If one stays within the area of the non-altered tones, then the resulting tones of this heptachord are the descending succession A, G, F, E, D, C, B:



A heptachord, however, does not give the experience of being a complete scale. The octave is required for this. Pythagoras gave special prominence to the octave as the compass and goal of the scale, and his followers called it 'the most perfect of all intervals'.[4] To make the octave part of the true-tone scales, Pythagoras is supposed to have expanded the true-tone heptachord of Terpander to the true-tone octachord by separating the two connected Dorian tetrachords in the middle by a major second and also by separating the unified third step, trite-paramese, by a true minor second (limma).[4, 11a] Together, the two tetrachords made up the Pythagorean (true) Dorian octachord which, when not including altered tones, probably ran from e to E (see diagram at top of p. 21).

Drawing on other sources, Pfrogner[7] writes that Terpander expanded the Apollonian heptachord to the octave by adding an uppermost tone and that Pythagoras then arranged it in the form presented above.

The importance of the octave as an interval which contains the scale can be clearly experienced by us today. Play or sing a scale in either ascending or descending direction and pause on the third, fourth, fifth, sixth or seventh. A temporary point of rest may be found at the fourth or fifth, yet one still has the feeling that the scale is incomplete. It is only when the octave is reached that one experiences it

Upper Dorian tetrachord (= lower of Terpander)				**Lower Dorian tetrachord** (= higher of Terpander)			
Nete	Paranete	Trite	Paramese	Mese	Lichanos	Parhypate	Hypate
e	d	c	H	A	G	F	E

Interval	e–d	d–c	c–H	H–A	A–G	G–F	F–E
Name	true whole tone	true whole tone	limma	separating true whole	true whole tone	true whole tone	limma
Ratio	9:8	9:8	256:243	9:8	9:8	9:8	256:243

to be complete. With descending scales, which have a major seventh on the seventh step, the listener can have the feeling that he or she would sink into an abyss if the encompassing octave was not reached.

Concerning the experience of the octave, Rudolf Steiner said on 7 March 1923:

> . . . the link to the world [through musical experience] will come when the experience of the octave arises in the described manner. For then the musical experience will be proof to human beings that God exists, as they will then experience the 'I' twice—once as a physical, inner 'I', and secondly as a spiritual, outer 'I' . . . They will say to themselves: Once I experience my 'I', as it is on earth, in the unison prime, and then experience it again as it is in the spirit, this will be inner proof that God exists.[30]

The octave is an exceptionally important interval in the musical experience of Western humanity. Many scales are contained within the octave, and all intervals, which we seek to understand here, are contained in such scales. Each scale which progresses from a keynote or prime to the octave, whether it ascends or descends, is heard by most people to be an organic whole—with careful listening, even the twelve-toned chromatic scale. The whole interval structure of a scale can thus be considered to be an organic structure with its own laws that proceeds from the octave. The form principles, as they will be developed later on, take this structure into account.

Pythagoras and his followers discovered, amongst other things, the harmonic, geometric and arithmetic means in music,[5] which, in accordance with Ernst Bindel, can be expressed in a single formula:[11c]

arithmetic mean	$m = \frac{1}{2}(a + b)$
geometric mean	$\log m = \frac{1}{2}(\log a + \log b)$
harmonic mean	$1/m = \frac{1}{2}(1/a + 1/b)$
or commonly:	$f(m) = \frac{1}{2}(f(a) + f(b))$

The harmonic mean and the arithmetic mean both divide the octave into a fifth and a fourth. If the octave is divided by both means, two perfect fourths are created, separated by a major second in the middle of the octave:

arithmetic mean

harmonic mean

If one adds the major second to the fourth one gets the fifth.

The Greeks knew that the geometric mean divides the double octave into two octaves 4:2:1, and the ditone (true major third) into two major seconds 81:72:64.[5] Dividing the single octave by the geometric mean one gets the equal-tempered tritone 2:1.4142 = $2\sqrt{2}:1$, which first came to be commonly known through the introduction of equal-tempered tuning and also sounds genuine to the ear.

These three means create the musical intervals and thus are among the archetypal phenomena. Gioseffo Zarlino (1517–90) found that with the fifth the harmonic mean gives rise to the major and the arithmetic mean the minor triad. His method is interesting. Instead of using the whole-number overtone intervals, he used the fractions of the division of string length. For the major triad he used the string sections 1/4, 1/5 and 1/6. When he presented 1/5 as the centre between 1/4 and 1/6 this appeared as the harmonic mean. As a result, he called the just major triad 'armonica'.[19] If, however, we consider the same triad as arising from the overtone intervals 4, 5 and 6, then 5 is calculated to be the arithmetic mean between 4 and 6. This is why the same just triad was described as 'divisione armonica' by Zarlino and arising through arithmetic division by us. The same applies to the just minor triad, which Zarlino called 'divisione aritmetica' whilst we describe it as arising from harmonic division. The reciprocity of undertone or overtone interval numbers and the division of string length is also evident from the formula for the arithmetic and harmonic means given above, as well as the monochord formula (see Appendix 1, p. 171).

Zarlino and his followers, e.g. Moritz Hauptmann (1792–1868), apparently only used arithmetic and harmonic division for the triads and not for the structure of complete scales as is done in the following chapters. It was mentioned above that we experience the scale contained within the octave to be an organic whole. The question of how the intervals of a given scale are produced will now be investigated by progressing from the octave and showing how one or more of the three above-mentioned means function. The laws found for the structure of the scales will be called the form principles of the scales. These form principles will serve to show that all intervals that sound genuine to the ear are closely connected with the harmonic, geometric and arithmetic means proceeding from the interval of the octave.

7 The Form Principles of the Just Scales

The just major scale

To avoid misunderstandings, let us mention again that there are 'two' methods of calculation: either according to the whole numbers of overtone or undertone partials (interval proportions) or according to the fractions resulting from the divisions of a string length. For the sake of simplicity, the calculations in Tables 3–13 are based on the former, and the first tone of the scale.

Firstly we will consider the just major scale as it is presented in Table 3. There we can see how the interval degrees of the just major scale are produced by firstly dividing the octave into the dominant and subdominant through the harmonic and arithmetic means (I, V, VIII; I, IV, VIII). This is called primary division. The characteristic nature of the just major scale is, however, only created by a secondary division using the arithmetic mean only. The arithmetic mean between the fundamental (8) and the fifth (12) gives the 10th overtone, creating the proportions of the just minor and major thirds 12:10:8. The ninth overtone is obtained as the arithmetic mean between the fundamental (8) and the just major third (10), i.e., the minor and major second 10:9:8.

Only just major thirds are used in the just major scale. The 27th overtone of the scale's fundamental ($13\frac{1}{2}$) also produced by the harmonic mean between the subdominant ($10\frac{2}{3}$) and the octave (16)), being true major sixth 27:16, would together with the fourth degree ($10\frac{2}{3}$) create an unwanted true major third 81:64. The sixth degree of the scale is therefore produced by the arithmetic mean between the subdominant and the octave: $16:13\frac{1}{3}:10\frac{2}{3}$. This new tone $13\frac{1}{3}$ is the just major third (i.e., the fifth overtone) of the subdominant and is nowhere to be found in the overtone or undertone row of the scale's fundamental. Thus a foreign tone is incorporated as the sixth degree into the just major scale in order to obtain a just major third that is above the fourth degree.

The form principles of the just major scale thus are harmonic mean and arithmetic mean divisions. According to our method of calculation, the arithmetic mean must be considered to be characteristic for this scale. This division gives the perfect consonances and the imperfect dissonances of this scale. All of these intervals are aurally experienced to be genuine. The temporary conclusion may thus be made that intervals created by arithmetically and harmonically dividing the octave are experienced as genuine.

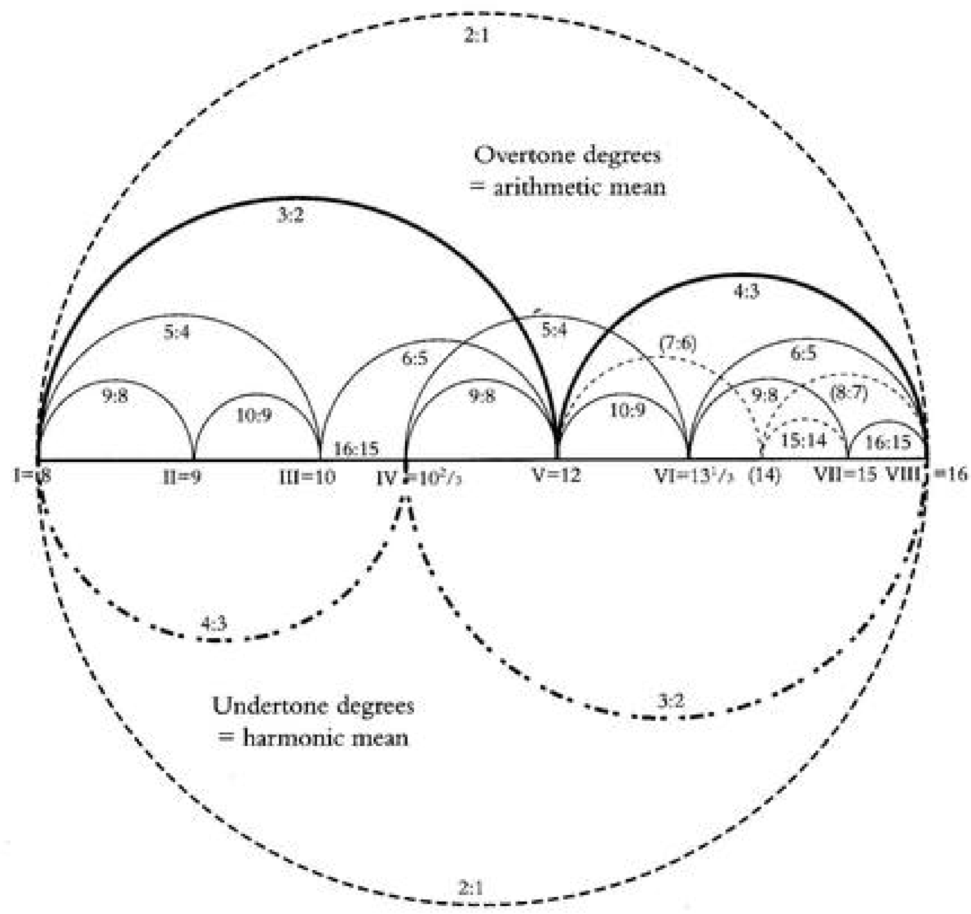
In harmony, intervals are combined in triads. The just major scale contains the following triads:*

three just major triads 6:5:4 on fundamental, dominant and subdominant;
two just minor triads $7\frac{1}{2}:6:5$ on third and sixth degree;
a diminished triad $10\frac{2}{3}:9:7\frac{1}{2}$ on seventh degree produced by a true minor third and a just minor third ($16:13\frac{1}{2}$ and 6:5);
a minor triad $13\frac{1}{3}:10\frac{2}{3}:9$ on second degree, which is harmonically intolerable, because it is contained by what Helmholtz called the 'grave fifth' $20:13\frac{1}{2}$.[13]

On a keyboard instrument tuned to the just major scale the latter triad can easily be heard. The fact that it is harmonically intolerable may be the reason why the laws of harmony strictly require that the triad on the second degree may only be used in its inversions and never in root position. Complications are met with when the harmony of a composition moves to the relative minor key because this chord, on the second degree in the major, is now on the fourth degree (the first degree of the relative minor is the same as the sixth degree of the major key) and therefore immediately comes to the fore as part of the cadencial sequence I, IV, V, I.

* This can be aurally observed if one tunes a keyboard instrument to a just major scale.

Table 3
The form principles of the just major scale*



Interval mean		Cents	Degree	New scale-degree	Cents to prime
harmonic mean of octave	2:1	1200	16:8	10⅔ = IV = fourth	498.045
arithmetic mean of octave	2:1	1200	16:8	12 = V = fifth	701.955
arithmetic mean of fifth	3:2	701.955	16:10⅔	13⅓ = VI = just major sixth	884.359
arithmetic mean of fifth	3:2	701.995	12:8	10 = III = just major third	386.314
arithmetic mean of just major third	5:4	386.314	10:8	9 = II = major second	203.910
arithmetic mean of fourth	4:3	498.045	16:12	14 unused (very small just seventh)	(968.826)
arithmetic mean of augmented whole tone	8:7	231.174	16:14	15 = VII = just major seventh	1088.269
arithmetic mean of too small just minor third	7:6	266.871	14:12	13 unused (13th overtone)	(840.528)

primary division:	arithmetic mean	———	harmonic mean	— · — · —
secondary division:	arithmetic mean	———	arithmetic mean —unused degree	---------------

* The table shows the proportions of the intervals, so the arithmetical proportions are not correct.

In singing and instrumental practice it is, of course, possible to get around this problem by making the tone on the fourth degree of the relative minor scale relate to the fundamental of the major scale in the proportion 10:9, i.e., somewhat lower than the second degree of the major. Proportionally this tone is the harmonic mean of the just major third 5:4 (20:16). The second degree of the major scale (formed by the arithmetic mean) is therefore replaced in the following way:

Degree:	III		II		I		In the relative minor the
Arithmetic mean:	20	:	18	:	16		18th partial is
Proportion:		10:9		9:8			discarded and
Harmonic mean:	20	:	17.78	:	16		17.78 is used
proportion:		9:8		10:9			

The triad on the fourth degree is now contained within a perfect fifth. Helmholtz's 'grave fifth' in the just minor scale has moved from the cadencial triads but is by no means removed from the scale. It is merely displaced and now comes between the seventh and fourth degrees. The just minor scale now includes a tone that is not contained in the corresponding just major scale, and is therefore, in the strict sense of the word, no longer in the relative key.

If one considers the just major scale in relation to the overtone row of its prime, it is not difficult to see that the just major third 5:4 is the reason for the appearance of the 'grave fifth' ($20:13\frac{1}{2}$ or 40:27) and its inversion the 'acute fourth' ($13\frac{1}{2}:10$ or 27:20).[13] Because the 27th overtone, or its lower octave $13\frac{1}{2}$, is not used for the sixth degree in the scale, the 'grave fifth' moves to between the sixth = $13\frac{1}{3}$ and second = 9, instead of between the third = 20 and the sixth = $13\frac{1}{2}$ degrees of the scale. Metaphorically speaking, the 'grave fifth' is the inevitable shadow side of the just major third. This fact should be regarded as belonging to the nature of the just major and minor scales.

The diatonic just minor scale

As with the just major scale, primary division of the octave into dominant and subdominant is by the arithmetic and harmonic means. As we have seen, the tone placed on the fourth degree of the just minor scale is not the same as the tone of the second degree of the relative major scale and is therefore foreign to the latter scale.

Secondary division is different. In this scale the characteristic intervals are produced by the harmonic mean (see Table 4):

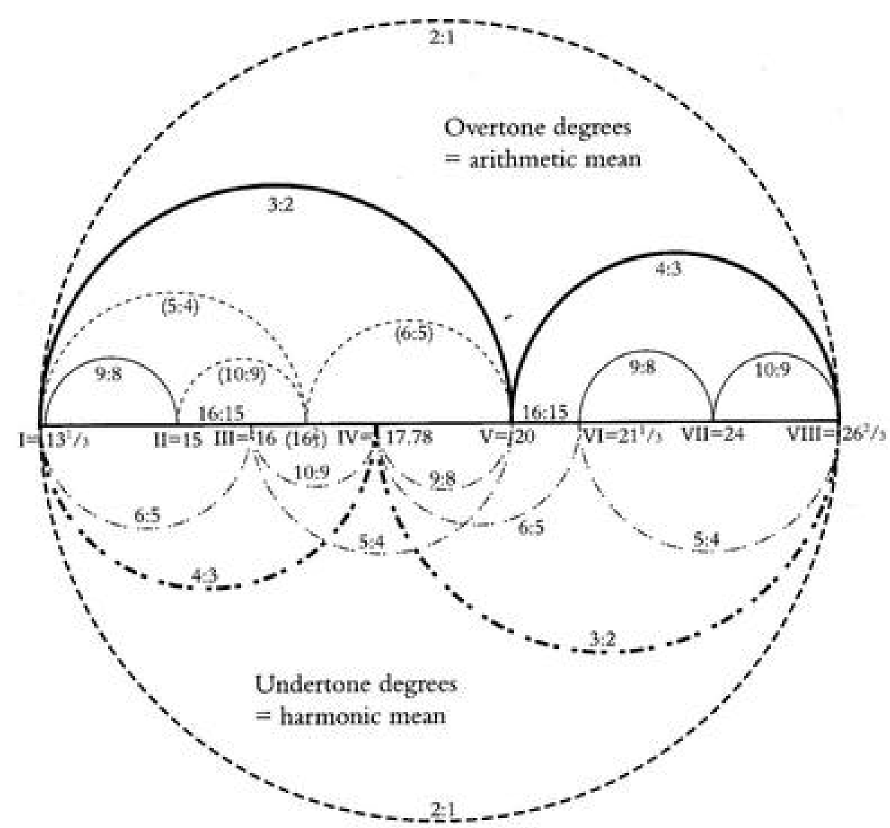
harmonic mean between prime = $13\frac{1}{3}$ and fifth = 20 produces the just minor third = 16;
harmonic mean between fourth = 17.78 and octave = $26\frac{2}{3}$ produces the just minor sixth = $21\frac{1}{2}$.

Division by harmonic mean goes no further. The major seconds at both ends of the scale need to be created by using arithmetic mean again:

arithmetic mean between minor sixth = $21\frac{1}{3}$ and octave = $26\frac{2}{3}$ gives the minor seventh = 24;
arithmetic mean of prime = $13\frac{1}{3}$ and the = 20 gives the just major third = $16\frac{2}{3}$ (not used in this scale);
arithmetic mean of prime = $13\frac{1}{3}$ and just major third = $16\frac{2}{3}$ gives the just major second = 15.

The form principle of the diatonic just minor scale has the same primary division as the just major scale. But whereas the major scale gains its character by secondary division through the arithmetic mean, the minor scale's character mostly comes about through secondary division by the harmonic mean. Not all intervals of the just minor scale are given by the harmonic mean; three of its major seconds are produced by tertiary division with the arithmetic mean.

Table 4
The form principles of the just minor scale*



Interval mean		Cents	Degree	New scale-degree	Cents to prime
arithmetic mean of octave	2:1	1200	26⅔:13⅓	20 = V = fifth	701.955
harmonic mean of octave	2:1	1200	26⅔:13⅓	17.78 = IV = fourth	498.045
harmonic mean of fifth	3:2	701.955	20:13⅓	16 = III = just minor third	315.641
harmonic mean of fifth	3:2	701.995	26⅔:17.78	21⅓ = VI = just minor sixth	813.686
arithmetic mean of fifth	3:2	701.955	20:13⅓	16⅔ unused (just major third)	(386.314)
arithmetic mean of just major third	5:4	386.314	26⅔:21⅓	24 = VII = just minor seventh	1017.596
arithmetic mean of just major third	5:4	386.314	16⅔:13⅓	15 = II = major second	203.910

primary division: arithmetic mean — harmonic mean
secondary division: arithmetic mean — harmonic mean
arithmetic mean —unused degree

* The table shows the proportions of the intervals, so that the arithmetical proportions are not correct.

8 The Seven Aulos Modes of Ancient Greece

Other just scales also exist. Though hardly used in Western music over the last four to five hundred years, they have been known from ancient times and have been in use to this day amongst the people of Asia, Greece, the islands west of Scotland, north America, etc. These scales, here called the seven aulos modes* of ancient Greece, are called 'harmoniai' by their rediscoverer, musicologist Kathleen Schlesinger.† As curator of old musical instruments in the British Museum she noticed that the finger-holes of the Greek aulos appeared to be equidistant. Thanks to her scientific training she knew that such instruments could not produce the scales and intervals in use today. The scales must therefore have been different. The types and forms of scales were established by Schlesinger in more than 25 years of research and published in 1938 in her book *The Greek Aulos*.[8]

One of her colleagues, Isabel Bonar Dodds, heard and collected just modally sung folk songs in the Gaelic islands of Britain. The author has heard the four-part just modal unaccompanied choir of the Ballet Folklorico de Mexico, which were of convincing beauty and richness. At the World Exhibition in New York in the 1960s, African drummers played instruments tuned exactly the same as the Mixolydian Moon mode 14/14. In the south-west of the USA interesting modal Indian flutes are still to be heard today. A few years ago, the author heard two modern just modal flutes, one originating in Korea, the other a Greek church flute, each tuned to a different mode. (Both of these flutes and a third as well, will be discussed in the chapter on the quality of single tones.) Such examples can be added to at will, but these should suffice to illustrate the wide geographical and historical spread of modal tuning. This alone may inspire interest in the aulos modes.

For the present-day musician, however, there is a much more important reason to occupy oneself with the modes. Two of the seven modes are the foundation for three of our present scales. The just major scale has arisen from the Hypolydian Mars mode, and the harmonic and melodic minor scales from the Phrygian Venus mode.[8] In chapter 9 an attempt will be made to show that these three scales can only be understood in the light of the underlying principle of the named modes. The Dorian aulos Sun mode, although it has no connection to the scales in present-day use, is also important and will be considered in Part Three.

Below, the modes are given with their connections to the planetary incarnations of the earth as given by Rudolf Steiner,[31] and the Ptolemaic world system.[32] The connections with the planets were also mentioned by Nicomachos,[4] though he limited himself to the true scales. Both relationships were discussed by Schlesinger with her students, but not mentioned in her book. Concerning this, Elsie Hamilton wrote the following:[9]

> One is often asked how one knows that, for example, the planet Saturn has a relationship to the Hypodorian mode (Saturn mode); the planet Jupiter with Hypophrygian mode (Jupiter mode); the planet Mars with the Hypolydian mode, etc. To answer this, one would have to go back to the Ancient Chaldean period, when the wisdom of the gods was still imparted to the initiates of old and from this source Pythagoras also received the knowledge which he was able to impart to his pupils in his esoteric school in Greece in about 600 BC. Traces of this teaching might perhaps be

* Mode means scale or key and in the present context has the same meaning as scale.

† Very little is known about the biography of Kathleen Schlesinger. She was born near Belfast in Ireland on 27 June 1852 and died in London on 16 April 1953. The greater part of her life was spent in England, where amongst other things she also worked at the British Museum. She was a musicologist and a Fellow of the Liverpool University Institute of Archaeology. In 1924, having become a member of the Anthroposophical Society in the meantime, she was asked by Rudolf Steiner to come to the Goetheanum in Dornach. Unfortunately this did not prove possible. Apart from smaller publications, she wrote *The Instruments of the Modern Orchestra* (2 volumes, London 1910) and her main work, *The Greek Aulos* (London: Methuen & Co. 1938).

> found in ancient Greek writings; but Kathleen Schlesinger may not have wished to mention this connection of the Greek modes with the planets in her monumental work, *The Greek Aulos*, feeling she was up against the most hardened and materialistic thinkers of the day who might have considered such knowledge as mere charlatanism, which would have been highly distasteful to someone of her scientific way of thinking. For such ancient teachings could only be handed down to us by tradition . . . (p. 25)

The structure of the modes is complicated and much patience is needed, entering into the thinking behind them, if we are to understand them. The structural principle becomes apparent when one reproduces them on a monochord.[8, 9, 11a, 25] This is one of the reasons why the construction of a monochord is given in Appendix 1. It is, however, also possible to understand the structure of the modes by means of acoustic and mathematical considerations.

Modes based on a common fundamental—the generic tone being different for each mode

In chapter 3 it was shown how the overtone row of a fundamental arises (Table 1). If one divides a monochord string into 16, 18, 20, 22, 24, 26 or 28 equal segments, then on the first segment the 16th or 18th etc. overtone is heard. These seven overtones and their octaves are the seven generic tones of the seven modes. The generic tones given at the top of p. 29 on the two string fundamentals c = 128 Hz and fau = 176* are mentioned because, according to Schlesinger, they have been known and in use from ancient times.

For each of the seven generic tones, the monochord string is subdivided into the number of equal parts that correspond to the generic tone's overtone number, e.g. 16, 18, etc. The generic tone then sounds on the first, uppermost segment, and as one progressively plays one, two, etc. segments of the subdivision, the descending undertone row of the generic tone is heard. The example of a 22/22 subdivision may serve to explain this.

If string length 1 is subdivided into 22 equal lengths, the first segment 1/22, gives the generic tone, prime or Arche of the mode. If two segments (2/22) are played, the length of string being doubled, the lower octave of the generic tone is heard. Three segments give the lower fifth, four the double octave, five the descending major third, etc., until with 22 segments the fundamental of the string is reached. The same applies to the undertone rows of the other six generic tones, with the undertones taken to the point where they reach the fundamental of the string: 16/16, 18/18, etc.

Comparison of this undertone row with Table 1, where the undertone row of c = 128 Hz is given, will show that it is longer. It goes to the 22nd while Table 1 only goes to the 16th partial. There we see a scale of seconds appear first between the 8th and 16th partials. A mode is a scale of seconds contained within an octave. The tonal material of the mode begins on the undertone, which is half the partial number of the generic tone, and progresses in descending direction until the single unit, i.e., the fundamental of the string, is reached. As, after the 16th partial, the proceeding undertones produce intervals smaller than a just minor second = 16:15, the modes that lie below the 16th partial use the doubled undertone partials of the diatonic octave (8th to 16th partial) and omit the undertones that lie in between (see diagram on p. 31). The sequence of intervals is the same for each generic tone. The tones, however,

* The tone 176 Hz is the 11th overtone of $_{2}C$ = 16 Hz and is here called fau. Schlesinger consistently called this tone f = 176 Hz in her book, but apparently fau when teaching. The name f = 176 Hz seems rather unfortunate as this tone will then easily be confused with the subdominant, i.e., the fourth of the tone C, which only appears in its undertone row (see Table 1). In contrast to this, fau = 176 Hz is the 11th overtone of $_{2}C$ = 16 Hz, its relationship to c = 128 Hz being 11:8. It seems sensible therefore to differentiate this tone from the tone F by the choice of term.

On c = 128 Hz:

Overtone+	Proportion+	Generic tone+	Hertz	Mode
16	1/16	c^4	2048	Hypodorian
18	1/18	d^4	2304	Hypophrygian
20	1/20	e^4	2560	Hypolydian
22	1/22	fau^4	2816	Dorian++
24	1/24	g^4	3072	Phrygian
26	1/26	high a♭^{4}	3328	Lydian
28	1/28	low b♭^{4}	3584	Mixolydian

On fau = 176 Hz:

Overtone+	Proportion+	Generic tone+	Hertz	Mode
16	1/16	fau^{4}+++	2816	Hypodorian
18	1/18	high true f♯♯^{4}	3168	Hypophrygian
20	1/20	high a^{4}++++	3520	Hypolydian
22	1/22	b^4	3872	Dorian
24	1/24	high c^5	4224	Phrygian
26	1/26	low d^5	4576	Lydian
28	1/28	e♭^{5}	3584	Mixolydian

+ In Tables 34 and 35 of Appendix 1, these numbers are doubled, i.e., taken an octave higher, in order to gain a double octave for each mode.

++ The generic tone of the Hypodorian mode with the string fundamental fau = 176 Hz has the same Hertz as the generic tone of the Dorian mode, with string fundamental c = 128 Hz.

+++ Present-day international concert pitch.

are different.* For example, the mese† always comes on the same interval proportion but the tone of the mese is different for each mode and corresponds to the mode's generic tone. The generic tone itself is determined by the pitch of the string.

To make clear distinction between the intervals and generic tones of each mode, Schlesinger developed the following fraction terminology. The denominator of the fraction is the number of the generic tone, called the modal determinant by Schlesinger. This gives the overtone partial of the generic tone as a proportion to the common string fundamental and moreover indicates the number of equally large segments into which the string or aulos is to be subdivided. The numerator of the fraction, on the other hand, gives the undertone partial in question (see table of the 22/22 Sun mode on page 30). Schlesinger named the fractions as a whole 'modal ratio numbers'.

From the above we might suppose that the aulos modes are descending scales. This is by no means the case. As Schlesinger expressly maintained, the modes are *ascending* scales. Having described how the pitches of the different partials of a mode are to be found on a monochord string, Schlesinger wrote (*The Greek Aulos*, pp. 10–11):[8]

> The constitution of the Harmonia or modal octave calls for further explanation. The procedure just described, which effects the genesis of the mode, has been explained upon the A string because

* Compare Table 34 and 35 in the Appendix, where the Hz of the numerators of the fundamental c = 128 Hz are given for the divisions of the monochord board. They clearly show that each mode contains other tones.

† The lower octaves of a generating tone sounding upon the numerators 2, 4, 8 etc. are called 'mese'.

> it cannot be achieved in practice upon the aulos; for the hole representing the position of the Arche would have to be placed upon the little straw mouthpiece itself, and the second and third segments might likewise fall either upon it, or so close to it that no sound could be elicited. It is the *ascending* Harmonia alone that comes to birth upon the aulos . . . .

She explained shortly afterwards how this was to be understood (p. 12):

> From the genesis of the modal material, of which we have now obtained some idea, the Harmonia, or octave scale, of the mode is derived; it begins on the note of the whole string [lowest tone] . . . and extends to the octave *above* . . . (Author's italics)

For these reasons, which have consequences in practical music making, the modes will be presented here as ascending scales, even though they are often presented in the relevant literature as being descending.[11a,25]

The lowest tone, the common fundamental, is therefore also the first tone of a mode, and its intervals run in the opposite direction to the undertone row of the generic tone, i.e., in ascending order. This will be elucidated with the example of the 22/22 Sun mode:

Degree	I		II		III		IV		V		VI		VII		VIII
Tone	22/22		20/22		18/22		16/22		15/22 or 14/22		13/22		12/22		11/22
Interval		11:10		10:9		9:8		16:15 or 16:14		14:13		13:12		12:11	

Every octave section of the modes contains nine tones. However, only eight can be used in a diatonic scale. One tone is therefore left out in each mode. In the Hypodorian and Hypophrygian modes the 14th, in the Mixolydian the 15th undertone is left out. In the remaining modes, the player is given the choice of using either the 14th or the 15th undertone, but must stay with the chosen tone for the whole of the composition. The Greeks divided their scales into tetrachords, and the interval structure of these

	First tetrachord				Second tetrachord			
Degree	I	II	III	IV	V	VI	VII	VIII
Hypodorian Saturn++	16+/16	15/16+++	13/16	12/16	11/16	10/16	9/16	8+/16
Hypophrygian Jupiter	18/18	16+/18	15/18+++	13/18	12/18	11/18	10/18	9/18
Hypolydian Mars	20/20	18/20	16+/20	14/20 or 15/20	13/20	12/20	11/20	10/20
Dorian Sun	22/22	20/22	18/22	16+/22	14/22 or 15/22	13/22	12/22	11/22j
Phrygian Venus	24/24	22/24	20/24	18/24	16+/24	14/24 or 15/24	13/24	12/24
Lydian Mercury	26/26	24/26	22/26	20/26	18/26	16+/26 or 15/26	14/26	13/26
Mixolydian Moon	28/28	26/28	24/28	22/28	20/28	18/28	16+/28++++	14/28

+ Numerators 8 and 16 = mese. ++ Ptolemaic planetary sequence. +++ 14th undertone must be left out. ++++ 15th undertone must be left out.

tetrachords was what counted. Also, as the subdivisions between the generic tone and the fundamental of the string were different for every mode, the intervals of the modes proceeded in different sequences. Each mode had two tetrachords. The tetrachords, tones and interval structures of the modes were as shown at the foot of p. 30.

The modes as subspecies of a common generic tone and common mese—the generic interval being different for each mode

Until now the seven aulos modes have been presented as proceeding from a common fundamental. Each mode therefore had a different generic tone, with different tones making up the scale, the result being seven different possibilities for each tone, e.g. seven different Cs. This makes it impossible to play all modes on a single instrument. However, a further group of aulos modes can be created where this becomes possible.

If one replaces the common string fundamental with a common mese, all seven modes lie on the undertones of the same generic tone and each mode is therefore given its own fundamental. If one begins with the mese on the 8th or 16th undertone, each tone in the scale which follows is a fundamental of one of the modes. The interval between the fundamental and the mese is called the generic interval by Schlesinger. Each mode is then played from the undertone partial of the fundamental up to its higher octave. Schlesinger called the modes created in this way subspecies. According to her the generic intervals and the modal ratios of the six subspecies of the Hypodorian Saturn mode are as follows:[8]

			Generic Interval		
Mode	**Planet+**	**Modal Ratio**	Name	Interval	Cents
Hypodorian++	Saturn	16/16	prime/octave	1:1/2:1	0/1200
Hypophrygian	Jupiter	18/18	major second	9:8	203.910
Hypolydian	Mars	20/20	just major third	5:4	386.314
Dorian	Sun	22/22	modal tritone	11:8	551.318
Phrygian	Venus	24/24	perfect fifth	3:2	701.955
Lydian	Mercury	26/26	just sixth	13:8	840.528
Mixolydian	Moon	28/28	very small just seventh	7:4	968.826

+ Planetary sequence according to Ptolemy.[22]

++ Here for the first time, the connection between the Hypodorian aulos mode and the planet Saturn becomes clearer: the generating tone of this mode is the prime = the beginning, in Greek Kronos or Saturn.

All in all, the seven subspecies cover the interval of an octave and a very small just minor seventh. Table 5 shows the seven generic intervals between the seven fundamentals and the common mese c = 128, or, respectively, the common generic tone c^5 = 4096 Hz.

If one plays the modes as subspecies of a common mese, as shown in Table 5, a meaningful and significant connection shows itself. The tones of the second tetrachord repeat themselves as the tones of the first tetrachord of another mode. According to Elsie Hamilton, each tetrachord of the seven modes is also related to a planet,[9] with the numerator row 16, 15, 13, 12 related to Saturn, for example; 11, 10, 9, 8 to the Sun; 14, 13, 12, 11 to the Moon, etc. If one arranges the modes in accordance with the repetition of the tetrachords, the sequence shown at the top of p. 33 arises.

It should also to be mentioned that not only scales and tetrachords, but also the single degrees of the scale are related to the different planets according to their numerator. The first degree as generic tone and its lower octaves 2, 4, 8, etc. as mese are always related to Saturn.* A fitting association always applies

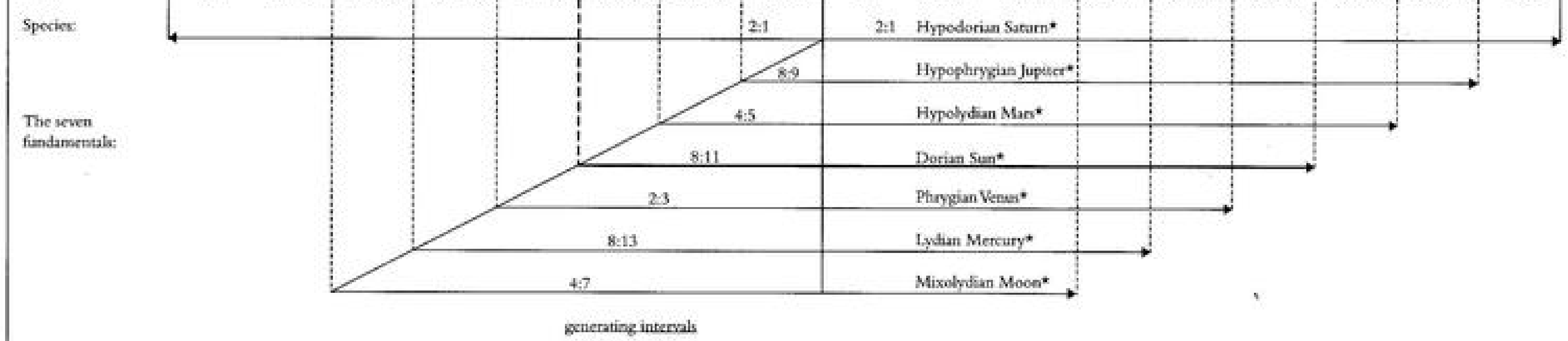
*See note ++++ to the table on p. 30.

Table 5

The seven aulos modes as species of the common mese c^1 = 256 Hz of the Hypodorian Saturn mode 32/32

Generic tone c^5 = 4096 Hz

Undertone ratio of the generating tone:	1/32 MESE	1/30	1/28	1/26	1/24	1/22	1/20	1/18	1/16 MESE	1/15	1/14	1/13	1/12	1/11	1/10	1/9	1/8 MESE
Modal ratio according to Schlesinger:	32/32	30/32	28/32	26/32	24/32	22/32	20/32	18/32	16/32	15/32	14/32	13/32	12/32	11/32	10/32	9/32	8/32
Tone:	true c	just d flat	just d	modal e	true f	modal geo	just a flat	true b flat	true c^1	just d flat^1	just d^1	modal e^1	true f^1	modal geo^1	just a flat^1	true b flat^1	true c^2
Hertz:	128	136.533	146.286	157.538	170.667	186.182	204.800	277.555	256	273.067	292.571	315.077	341.333	372.364	409.600	455.111	512



Fractions: According to our terminology the proportion between the undertone and the generating tone.
Modal ratio: According to the terminology of Schlesinger: numerator = number of equally long string segments = the undertone row partial number. Denominator = overtone partial number = generating tone.
* Planetary sequence according of Ptolemy.

THE SEVEN AULOS MODES OF ANCIENT GREECE

		First tetrachord				Second tetrachord				
		I	II	III	IV	V	VI	VII	VIII	
Hypodorian	Saturn++	32/32++	30/32	26/32	24/32	22/32	20/32	18/32	16/32	Sun+++
Dorian	Sun	22/22	20/22	18/22	16/22	15/22or 14/22	13/22	12/22	11/22	Moon
Mixolydian	Moon	28/28*	26/28	24/28	22/28	20/28	18/28	16/28	14/28	Mars
Hypolydian	Mars	20/20	18/20	16/20	15/20or 14/20	13/20	12/20	11/20	10/20	Mercury
Lydian	Mercury	26/26*	24/26	22/26	20/26	18/26	16/26	15/26or 14/26	13/26	Jupiter
Hypophrygian	Jupiter	18/18	16/18	15/18	13/18	12/18	11/18	10/18	9/18	Venus
Phrygian	Venus	24/24*	22/24	20/24	18/24	16/24	15/24or 14/24	13/24	12/24	Saturn

* Doubling of the numerator

++ Planetary sequence as in the evolution of the Earth according to Rudolf Steiner.[31]

+++ Planetary sequence according to the days of the week.

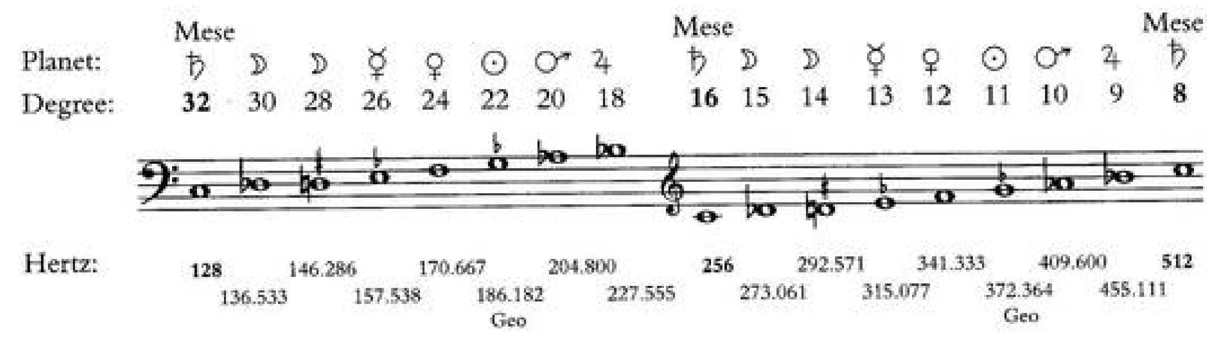
to the degrees which follow. This is important and will be dealt with in Part Three. The following overview results:

Degree:						Related planet
1, 2,	4,	8,	16,	32		Saturn
		9,	18,	36		Jupiter
	5,	10,	20,	40		Mars
		11,	22,	44		Sun
3,	6,	12,	24,	48		Venus
		13,	26,	52		Mercury
	7,	14,	28,	56		Moon
		15,	30,	60		Moon

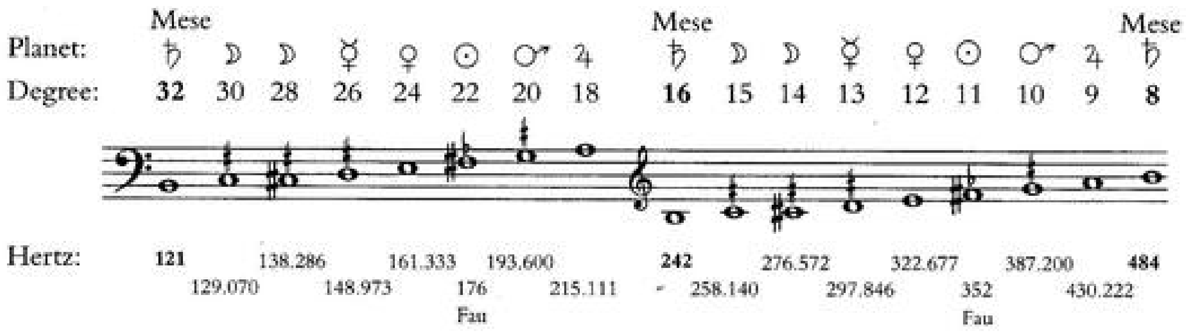
Only the Moon is related to two degrees of the scale, these being 14 and 15.

Two groups were given on p. 29, each with a different generic tone. Theoretically, the structure of the modes allows any tone to become the generic. This means that each of the planets can be related to any and every pitch. This will be illustrated in three examples given below. The generic tones c^5 = 4096 Hz and fau^4 = 2816 Hz have been chosen because we present the modes on the undertones of the former whereas Schlesinger and Hamilton mainly do so on the latter. (The reason for departing from Schlesinger's method of tuning is given in chapter 18.) The middle example with the generic tone b^4 = 3872 Hz is given because the undertone b = 242 Hz is the seventh degree of the equal-tempered scale of c = 128 Hz.

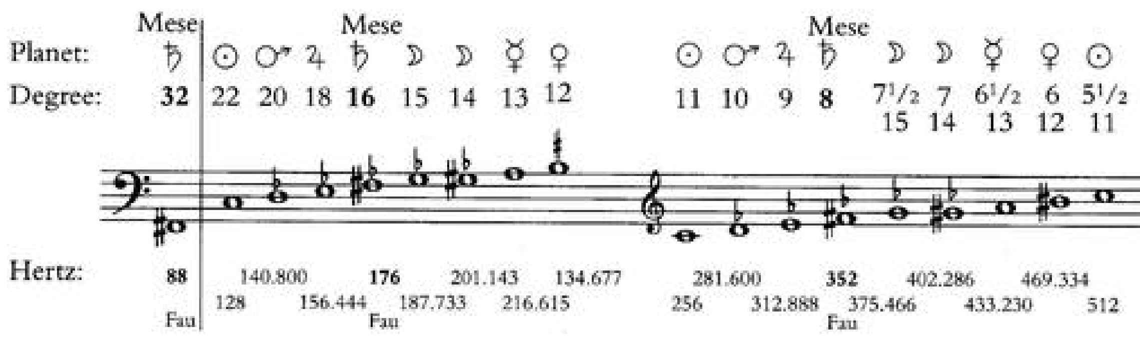
If c^5 = 4096 Hz is made the generic tone, then it and its lower octaves are Saturn tones, the remaining tones order themselves into the above arrangement, and the musical notation looks like this:



If one takes b^4 = 3872 Hz as the generic tone, then it and its lower octaves are Saturn tones, and the modal musical score (in which the tones c = 128 and c^1 = 256 Hz are absent) appears as follows:



If one uses fau^4 as the generic tone, then fau and its lower octaves are Saturn tones. In this case the tones c = 128 and c^1 = 256 Hz are present on the 11th and 22nd undertone partials of the generic tone and are accordingly related to the Sun. This row gives the following music score:



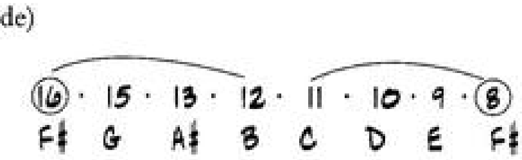
If, on the other hand, all the modes are played as species of a common mese, i.e., on the undertones of a single generic tone, then the tone/planet relation is the same for each of the modes.

A few quotations from Schlesinger and Hamilton will now be given that support the choice of generic tone in our presentation. Schlesinger communicated all connections between the modes presented here to her pupils, including Elsie Hamilton, in her lessons, but they are not given in her book. Elsie Hamilton therefore wrote:[9]

> Let me at once say that all the theoretical knowledge I possess has been imparted to me by her [Schlesinger] through our long and happy friendship over many years. All I can claim as my own contribution is the use I have made of these modes as a basis for modern composition, of which details have been given in Appendix 3 of *The Greek Aulos*.

As our presentation is extensively based on indications come down by word of mouth from Schlesinger's pupils, it seems appropriate to quote a few indications regarding the composition of the modes from Elsie Hamilton:[9]

Saturn (Hypodorian mode)



The second tetrachord 11 • 10 • 9 • 8 sounds rather like the major second scale of the Chinese, adopted by Debussy.

Jupiter (Hypophrygian mode)

18 · (16) · 15 · 13 · 12 · 11 · 10 · 9
E F# G A# B C D E

Mars (Hypolydian mode)

20 · 18 · (16) · 14 · 13 · 12 · 11 · 10
D E F# G# A# B C D

Both ratio 15 and 14 are never used together in the same mode, but there are two forms of the Mars mode, the earlier one with ratio 14 and a later one with ratio 15. This latter one is the prototype of our modern major scale, which has lost its original purity

Sun (Dorian mode)

22 · 20 · 18 · (16) · 14 13 12 11
C D E F# G# A# B C

This was the principal mode in the old [Dionysian] Greek musical system. [Note according to Schlesinger's manuscript: the Sun mode also has two forms, one with 14 and one with 15 (author's note)]

Venus (Phrygian mode)

24 · 22 · 20 · 18 · (16) · 15 · 13 · 12
B C D E F# G A# B

Again there are two forms of this mode. With ratio 15 it is the prototype of our harmonic minor scale; with ratio 14, of our [ascending] melodic minor scale:

24 · 22 · 20 · 18 · (16) · 14 · 13 · 12
B C D E F# G# A# B

Mercury (Lydian scale [mode])

26 · 24 · 22 · 20 · 18 · (16) · 15 · 13
A# B C D E F# G A#

Moon (Mixolydian scale [mode])

28 · 26 · 24 · 22 · 20 · 18 · (16) · 14
G# A# B C D E F# G#

The Moon scale (mode) must always have ratio 14, not 15. (Hamilton: *The Modes of Ancient Greece*, pp. 24–5)

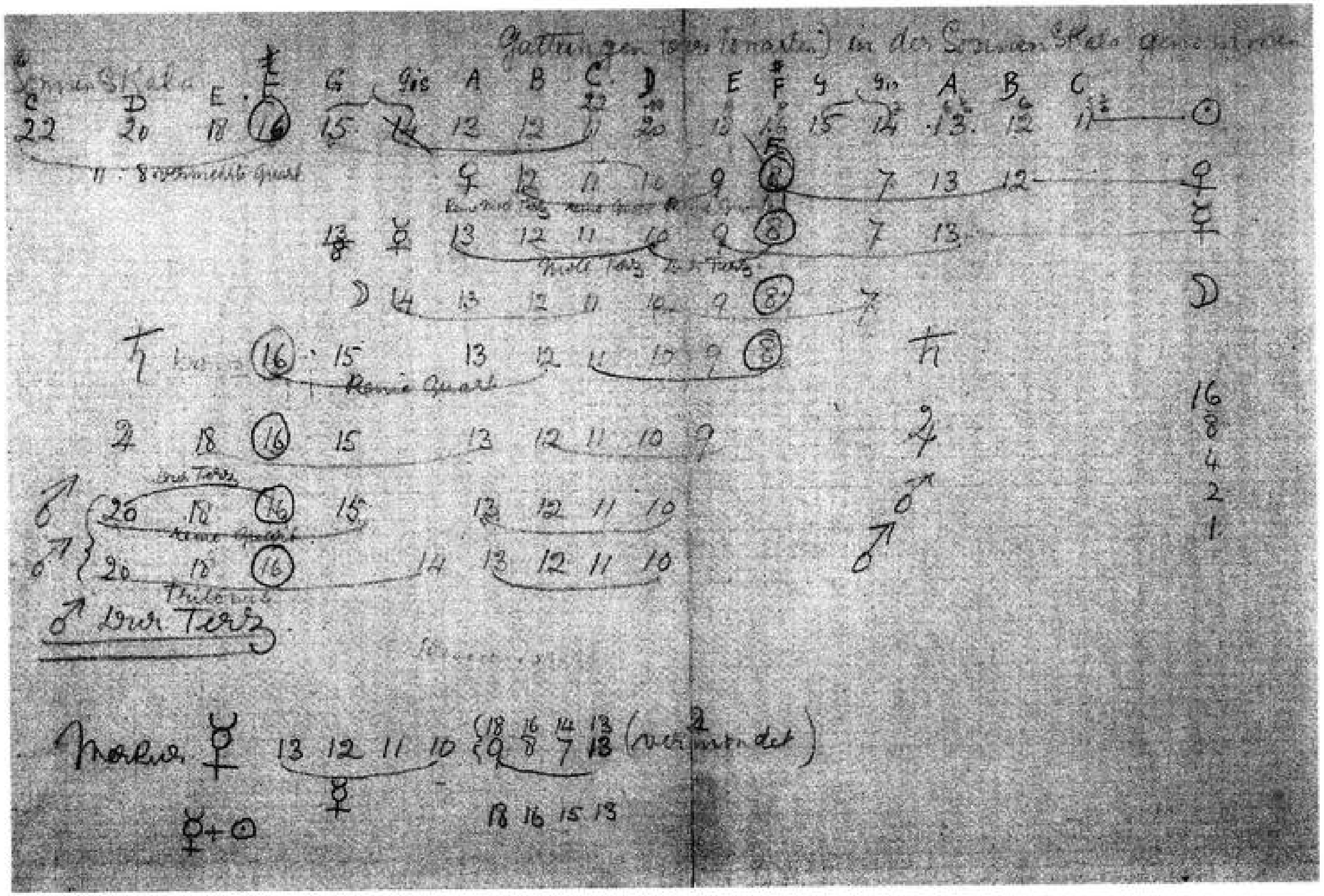
Apart from this, Schlesinger personally wrote out by hand for her colleague, eurythmy therapist Nanda Knauer, the degrees of the 22/22 Sun mode on the fundamental c = 128 Hz and mese fau = 176 Hz, together with all the species relating to it. Miss Knauer obligingly put this manuscript at the author's disposal. It is due to this that we are in the fortunate position to present an authentic facsimile of Schlesinger's own way of doing this.

A comparison of Schlesinger's way and Hamilton's shows that Schlesinger only gave ratio 14 for the Phrygian Venus mode. However, in the Sun Hymn she composed, which Knauer has also made available to us, she consistently used the tone g^1 = 375.466 Hz, i.e., ratio 15. This substantiates the possibility given by Hamilton to choose between ratios 15 and 14 in this mode. That the choice also applies to the Lydian Mars mode is evident from the last two lines of the manuscript. Especially noteworthy is the fact that in the lower octave of the Sun mode (line 1 in the facsimile), Schlesinger crossed ratio 14 out and drew the arc of the first tetrachord from the 14th as well as the 15th ratio. She thus indicated that ratio 15 may also be used in the Sun mode. The important possibility to play either ratio 14 or 15 in the Hypolydian Mars, Dorian Sun, Lydian Mercury and Phrygian Venus modes, as presented by us, has been comprehensively substantiated and authentically illustrated.

As a further characteristic of the modes, Schlesinger's manuscript shows the ascending direction of both scale and tetrachord (facsimile). This is especially significant as it means that the modes are the only scales known to us where the intervals get progressively larger in ascending direction (see table at the top of p. 33). This extension of the modal intervals reflects an extension of conscious awareness, becoming progressively more comprehensive, and the cyclic order of the tetrachords reflects an ever deeper penetration into cosmic interrelationships. Being pre-Christian scales, the modes mirror the way which the Dionysian mystics had to follow. They would first descend through the instinctive nature of evil within their own being—in the tonal material we find a related descending octave section in the undertone row of a generic tone with ever-diminishing interval sizes (see Table 1). Then the god Dionysus was found in the deepest tone (first tone of the mode) in order to ascend with the help of this initiate (Dionysus) into progressively greater and wider heights of initiation knowledge—ascending scales with steadily augmenting intervals and cyclic progressing tetrachords. In Schlesinger's complete presentation, the modes belong to the most momentous products of the Dionysian stream in music.

Modes and modal intervals played on a violin or monochord sound genuine to the ear. This means that it is also possible for us to find a musical relationship to the modes today.

Original note of Kathleen Schlesinger to Nanda Knauer: The seven aulos modes as species of the Sun mode 22/22, fundamental c = 128 Hz with the generic tone fau^4 = 2816 Hz, mese fau = 176 and fau^1 = 352 Hz (fau is written as F♯), NB: Gis = G♯



'Hymn to the Sun'

Melody by K. Schlesinger

From Sophocles' play *Antigone*, Choir 1, first verse.

Passed down by Nanda Knauer

Beam of the Sun, fairest light that ever dawned on Thebe of the seven gates, thou hast shone forth at last, eye of golden day, arisen above Dirce's streams!

9 Aulos Modes and Just Scales

The form principles of the Hypodorian Saturn aulos mode

To make a comparison between modes and just scales, the form principles of the aulos modes must first be found. If this is to be done on the same basis as the form principles of just scales as presented in chapter 7, the undertone ratios must be used instead of the modal ratios used by Schlesinger (see chapter 8). The undertone ratios describe the tones between which a particular mode lies just as accurately as the modal ratios.

When modes are derived from a common generic tone, the Hypodorian Saturn aulos mode 16/16 is the only mode that lies between two mese. The mese and the prime of this mode are the same and its tones follow the same sequence as the undertones of the generic C between the 8th and 16th partials. The structure of the Hypodorian Saturn mode may thus be brought into connection with the form principles of the undertone row of C. The example of the Hypodorian aulos mode given in Table 6, on p. 40, suffices for all of the modes as all modes contain the same intervals, only in different sequences.

Table 6 shows that all intervals of the modes are governed by a single form principle, namely, the primary, secondary and tertiary divisions of the octave by the harmonic mean. The aural observation as to the genuineness of all modal intervals is therefore also confirmed.

If we compare the Hypodorian Saturn aulos mode

Undertones:	1/16		1/15		1/13		1/12		1/11		1/10		1/9		1/8
Intervals:		15:16		13:15		12:13		11:12		10:11		9:10		8:9	

with the just major scale

Overtones:	1/8		1/9		1/10		$1/10\frac{2}{3}$		1/12		$1/13\frac{1}{3}$		1/15		1/16
Intervals:		9:8		10:9		$10\frac{2}{3}:10$		$12:10\frac{2}{3}$		$13\frac{1}{3}:12$		$15:13\frac{1}{3}$		16:15	

it is evident that with exception of partials 1/11 and 1/13, the two scales are a simple mirrored inversion of each other, the one built on the undertone row, the other on the overtone row of C.

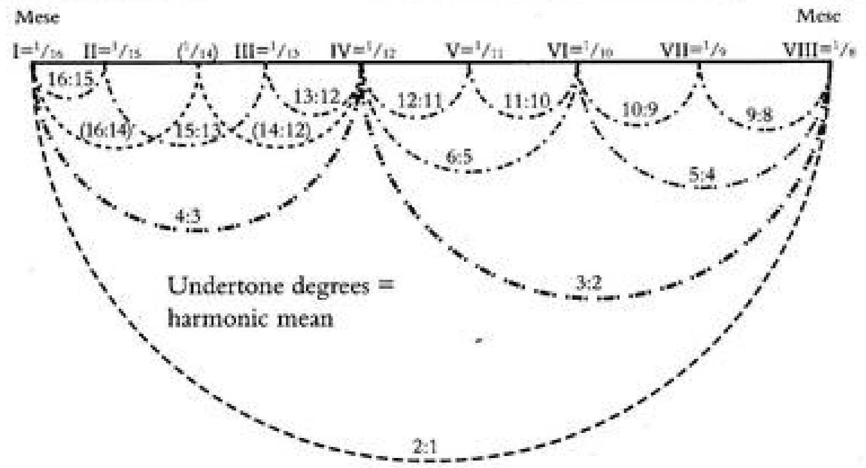
The Phrygian Venus aulos mode and its connection to the harmonic and melodic minor scales

It was shown in chapter 7 that the just minor scale stems from the sixth degree of the relative just major scale and, apart from of the fourth degree, contains the same tones.*

The seventh degree in the harmonic minor scale and the sixth and seventh degrees in the ascending melodic minor scale are raised by a semitone. These scales therefore arise from a compromise between their relation to the relative major scale and functional harmonic requirements. These requirements can be observed in the use of the minor scales in the sixteenth to nineteenth centuries where the gravitational power of the minor tonic is weaker than that of the major and is generally not maintainable as a tonic without raising the leading tone on the seventh degree and its preparation, the sixth degree, through borrowed tones from the tonic major. The designation 'relative minor' is therefore only truly

* In Table 3 it was shown how the 11th partial on the fourth degree of the just major scale, obtained through division by the arithmetic mean, is replaced by the tone $10\frac{2}{3}$ gained by primary division of the octave by the harmonic mean. With this single harmonic division the subdominant of the scale is placed on the fourth degree (see chapter 7, 'The just major scale').

Table 6
The form principles of the Hypodorian Saturn aulos mode*



Interval mean		Cents	Degree	New scale degree	Cents from mese 1/16	Cents from mese 1/8
Harmonic mean of octave	2:1	1200	1/8:1/16	1/12 = IV = fourth	498.045	701.955
Harmonic mean of fifth	3:2	701.955	1/8:1/12	1/10 = VI = just minor sixth	813.686	386.314
Harmonic mean of fourth	4:3	498.045	1/12:1/16	1/14 unused (augmented major second)	(231.174)	(968.826)
Harmonic mean of just major third	5:4	386.314	1/8:1/10	1/9 = VII = true minor seventh	996.090	203.910
Harmonic mean of just minor third	6:5	315.641	1/10:1/12	1/11 = V = modal tritone	648.682	551.318
Harmonic mean of too small just minor third	7:6	266.871	1/12:1/14	1/13 = III = modal major third	359.472	840.528
Harmonic mean of augmented just major second	8:7	231.174	1/14:1/16	1/15 = II = just minor second	111.731	1088.269

Primary division:	harmonic mean	— · — · —
secondary division:	harmonic mean	- - - - - -
	harmonic mean—unused degree	········

* The table shows the proportions of the intervals, so the arithmetical proportions are not correct.

valid for the diatonic minor. The modified harmonic and melodic minor scales therefore demand to be related to the ancient Greek aulos modes.*

What scale results when we produce the fifth degree of the Hypodorian Saturn mode with the arithmetic mean instead of the harmonic mean, i.e., when 1/11 is replaced by the upper dominant 3/32 of the mese? The tones and intervals are as follows:

* The relationship between the Phrygian aulos mode and the harmonic and melodic minor scales in use since the seventeenth century was observed by Schlesinger and Hamilton.[8,9] It is mentioned here as it solves the question of the relationship between the two forms of the minor scale and the relative major.

Undertones	1/16		1/15		1/13		1/12		3/32		1/10		1/9		1/8
Intervals		15:16		13:15		12:13		8:9		15:16		9:10		8:9	

If one plays or sings these tones, the impression is similar to that of the harmonic minor scale when beginning on the dominant.*

If the prime of the Phrygian Venus species of the Hypodorian Saturn mode is played from its subdominant, the following harmonic minor scale results:

Undertones	1/24		3/64		1/20		1/18		1/16		1/15		1/13		1/12
Intervals		8:9		15:16		9:10		8:9		15:16		13:15 (1½ tone)		12:13	

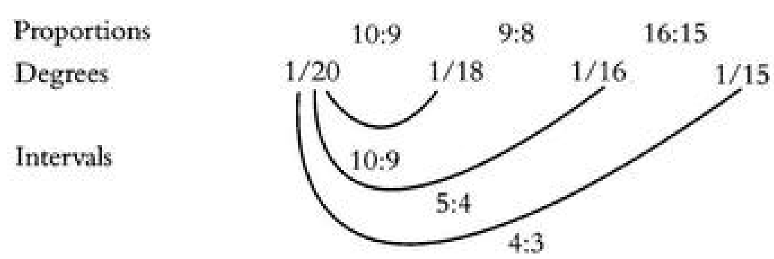
The objection may be made that the 12:13 interval at the end of the scale is bigger than a 15:16 just semitone. Mathematically this is correct. However, even though the ear hears it to be a bit larger, it experiences it as a semitone. We may therefore say that the Phrygian Venus mode with the 1/15 degree is practically identical to our present harmonic minor.[8, 9]

The melodic minor scale is also based on the Phrygian Venus mode but with the 1/14 degree instead of the 1/15. When played in ascending direction this version of the mode sounds very similar to our melodic minor.

If one reflects on the fact that the two forms of the Venus species of the Saturn mode form the melodic and harmonic minors and that the Venus species is based on the subdominant of the Saturn mode, it seems more appropriate to make a connection between the minor key and the subdominant of the major. Although foreign to modern musical thinking, this avoids pointing to an only fragmentary connection to the relative major and helps give an understanding for the different forms of the minor as individual scales.

Aulos Mars tetrachord and just major scale

Schlesinger wrote that the present just major scale most probably arose from the Hypolydian Mars aulos mode.[8] If one uses the 1/15 degree, the first (lower) tetrachord—the Mars tetrachord—consists of



This tetrachord (see Table 3) has the same intervals as the first tetrachord of the just major scale, though the first and second are swapped around: 9:8, 10:9, 16:15.

A mode always consists of two different tetrachords so that each interval only appears once, and the second tetrachord of the mode is therefore very different. But Schlesinger said that it was possible for the auletist to change the position of the lips on the tongue of the aulos' mouthpiece slightly and so play the

* From the point of view of the theory of harmony based on major/minor tonality, the harmonic minor suffers from the long-term ban on the augmented step between its sixth and seventh degrees, which makes it appear questionable whether it even obtains the rank of a genuine octave scale. However, it stubbornly maintains its position in the theory of music next to the melodic and diatonic minors, which would suggest that its fundamental composition is found in the scales of other periods and nations. It is therefore considered here as a fully valid minor scale.

lower tetrachord of a mode a second time a fifth higher on the dominant. In the case of the Hypolydian Mars mode, the Mars tetrachord would then appear twice. The mode then becomes a scale that is very similar to the just major scale where the second tetrachord, with the intervals 10:9, 9:8 and 16:15, is the same as the Mars tetrachord. The only difference thus lies in the first tetrachord. Placing one on top of the other, one sees that the first tetrachord of the Hypolydian Mars mode and the first tetrachord of the just major scale have the same second and fourth degrees, with the first and third different:

Degree	I	II	III	IV
Mars tetrachord	10	9	8	7½
Just major scale, first tetrachord	8	9	10	7½

The difference may be explained as follows.

In the case of the Hypolydian Mars mode, the generic interval is the major third 5:4 with the mese on the third degree of the mode. Because of the descending direction of the undertone row the major second between the second and third degree is larger than that between the first and second degree. In the case of the just major scale, the prime of the scale is also its generic tone and as its intervals are created by this tone's overtone row, so that the intervals get progressively smaller in the ascending direction, the major second between the first and second degree is larger than that between the second and third degree.

Mathematically this is due to the fact that in the case of the mode, the second degree is produced by the harmonic mean, whereas in the case of the just major scale it is created by the arithmetic mean. This difference is in keeping with the change in the division of the major third between the first and third degrees of the just major scale from arithmetic mean to harmonic mean in order to make the second degree a usable fourth degree in the relative minor scale (see chapter 7).

It may be seen from the above that the opinion that our present just major scale arises from the changed Hypolydian Mars aulos mode of the Dionysian musical stream in Greece is by no means unfounded.[8]

10 Twelve-toned Equal-tempered Scale

The equal-tempered scale will now be considered. In Table 7, on p. 44, the upper part shows how the division is usually presented.[11b] All intervals appear as a sum of the minor second $\sqrt[12]{2}$. All intervals therefore obey the same mathematical principle and one might consider that they all have the same aural quality. But to the ear the quality of the twelve intervals of this scale is completely different. The equal-tempered tritones, minor thirds and major sixths are experienced to be genuine, while the equal-tempered fifths, fourths, major thirds, minor sixths, major seconds, minor seconds and both sevenths sound false.

The lower part of Table 7 shows that the tritone, minor thirds and major sixths can also be produced by dividing the octave through primary and secondary division with the geometric mean.*

The geometric mean produces the equal-tempered tritone: 2:1.4142:1 between octave and prime; the equal-tempered minor third between the tritone and the prime: 1.4142:1.1892:1; the equal-tempered major sixth between the octave and the tritone: 2:1.6818:1.4142. The other intervals of this scale are produced by the accumulation of equal-tempered minor seconds, which are obtained either by dividing the octave by its twelfth root or by dividing the equal-tempered minor third by its cubic root: $\sqrt[3]{1.1892} = \sqrt[12]{2} = 1.0594$. The twelve-toned equal-tempered scale can, like the just scales, also be considered to be produced by two divisions. The primary division is by geometric mean, which produces the intervals that sound genuine to the ear. The secondary division is that of the octave by its twelfth root, or division of the minor third by means of the cubic root, to produce the equal-tempered minor second. But this equal-tempered minor second and the remaining seven intervals obtained by summation of the equal-tempered minor second all sound untrue, i.e., falsified to the human ear. This fact is commonly acknowledged and need not be discussed further.

The equal-tempered scale is a good example of how exactly the form principles of a scale mathematically support the aurally perceived genuineness or falseness of the intervals. The form principles are thus an invaluable help in gaining an intellectual understanding of the scales.

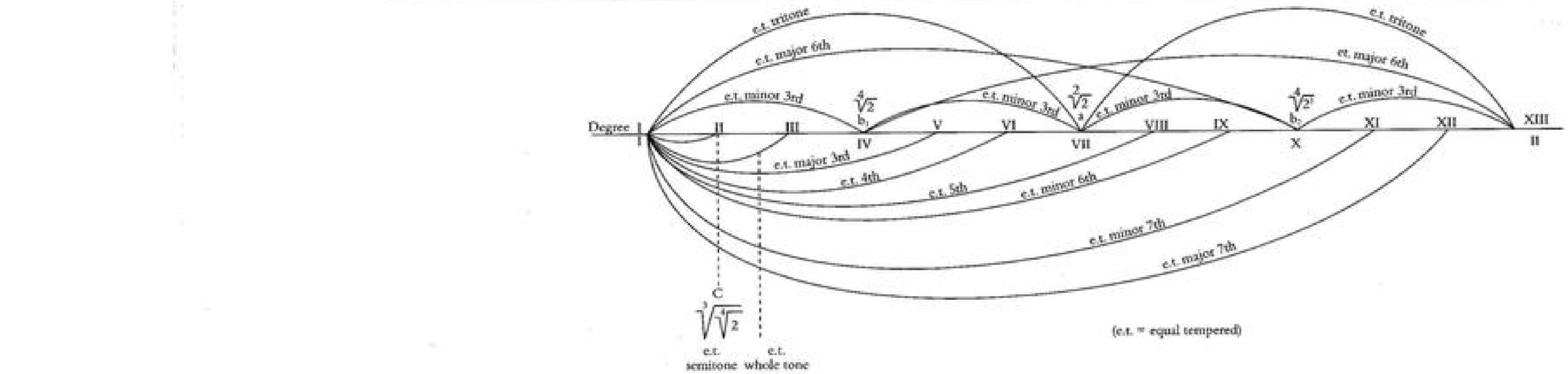
One may ask why the aurally perceptible genuineness of the equal-tempered tritone, minor third and major sixth has apparently remained unnoticed until now. A possible answer may be that the latter two intervals are usually paired with non-genuine fourths and fifths, their genuineness thus drowned out. It should, however, be stressed that equal-tempered tuning creates a genuine-sounding interval that appeared for the first time in Western music: the equal-tempered tritone. It is known that the tritone was called the *diabolus in musica* and that its use was expressly forbidden in strict counterpoint. If one examines the tritones in use at the time—the true augmented fourth and the true diminished fifth—this becomes understandable as both intervals with their exalting or also cramping properties forcefully demand resolution. The equal-tempered tritone sounds very different—peaceful though dissonant. It leaves the listener totally free to resolve it to the minor sixth, major third or to even leave it unresolved. With its non-urging sound it holds the balance, so to say, between the luciferic augmented fourth and the ahrimanic diminished fifth, the two *diaboli in musica*. Because of this, it may be permissible to call the equal-tempered tritone the Christian tritone. We will meet the tritone again as an important interval in the discussion of the scale of twelve fifths (see also chapter 13, and also chapter 21, p. 120).

As the twelve-toned equal-tempered scale has equally large intervals between the scale degrees, and the middle tone between each three neighbouring tones is the geometric mean of the other two, the mistaken idea has arisen that this scale is a consistent implementation of the geometric mean. However

* According to Boethius[3] the geometric mean is one of the fundamental principles of music. (Root-division is not a fundamental principle of music.)

Table 7

Genuine and false intervals of the twelve-toned equal-tempered scale,* calculated according to Ernst Bindel[11a]



Usual division: genuineness and falseness of the intervals is *not* evident from the numbers.

	1	$(\sqrt[12]{2})^0$	$(\sqrt[12]{2})^1$	$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^3$	$(\sqrt[12]{2})^4$	$(\sqrt[12]{2})^5$	$(\sqrt[12]{2})^6$	$(\sqrt[12]{2})^7$	$(\sqrt[12]{2})^8$	$(\sqrt[12]{2})^9$	$(\sqrt[12]{2})^{10}$	$(\sqrt[12]{2})^{11}$	$(\sqrt[12]{2})^{12}$	2
Degree:		i	ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii	xiii	

New division: form-principles of the scale. Genuiness and falseness of the intervals is evident from the numbers.

Upper bows: aurally genuine intervals

					$\sqrt[4]{2}$			$\sqrt[2]{2}$			$\sqrt[4]{2^3}$				
			ii	iii	b1	v	vi	a	viii	ix	b2	xi	xii		
Degree:	1														2
		i			iv			vii			x			xiii	

Lower bows: aurally false intervals

$\sqrt[3]{\sqrt[4]{2}}$

equal-tempered semitone

equal-tempered whole tone

Genuine intervals: $a = \sqrt[2]{2} = 1.4142$ = tritone = geometric mean of the octave

$b1 = \sqrt[2]{1.4142}$ = equal-tempered minor third = geometric mean between tritone and prime

$b2 = \sqrt[3]{2 \times 1.4142} = 2\sqrt{2.8284} = \sqrt[4]{2^3} = 1.6818$ = equal-tempered major sixth = geometric mean between octave and tritone

False intervals: $c = \sqrt[3]{2.1892} = \sqrt[9]{1.6818} = \sqrt[12]{2} = 1.0594$ = equal-tempered semitone = cubic root of the number of the equal-tempered minor third and the degrees derived from this: III, V, VI, VII, IX, XI and XII

* The table shows the proportions of the intervals, and is therefore not arithmetically proportionally correct.

the same phenomenon is to be found in the Siamese equal-tempered seven-degree scale, the Indonesian equal-tempered five-degree Salandro and the equal-tempered 19-degree supra-diatonic scale of Joseph Yasser.[34] Equal-tempered intervals are therefore by no means limited to the equal-tempered twelve-toned scale.

Only in the equal-tempered twelve-toned scale does the geometric mean division proceed from the octave as one of its form principles. This is not the case in Yasser's, the Siamese or Indonesian equal-tempered scales, as none of their intervals can be found by a geometric mean division proceeding from the octave. A scale totally produced by the form principle of the geometric mean would have to consist of the following intervals: 1 octave, 2 equal-tempered tritones, 4 equal-tempered minor thirds, 8 equal-tempered three-quarter tones, 16 equal-tempered three-eighth tones etc. and their combinations. None of the Western, Eastern or Arabic[35] scales known to the author show such a structure.

11 Greek Scale Names

Before considering the true-tone scales (church modes), it will be necessary to clarify the confusing use of Greek scale names. These names were first mentioned in chapter 8 and we will meet them again when considering the church modes below. The same names are used in both cases and brought into connection with the same planets, but they apply to different tones and to scales with completely different structures.

The two music streams of Ancient Greece—the Dionysian aulos and the Apollonian kithara streams—used the same names for each of their seven different scales and scale degrees. To make the confusion complete, the medieval Church used these names—though as far as we know without the connection to the planets—for still other scales and tones.[8,9,36]

To give the reader a guide through the labyrinth, Table 8 contains all the different scales, complete with names, scale degrees and interval proportions under the three headings Greek Dionysian, Greek Apollonian, and Church modes. In this book the scales are always given the names of the musical stream concerned (e.g. Dionysian aulos or Apollonian kithara), and in the case of church modes the Greek and not the church tetrachord names are used.

Table 8
The Greek names of the aulos modes, true scales and church modes

AULOS MODES			TRUE SCALES					
Greek Dionysian: structure is descending, scale is ascending. See chapter 8 and Table 5. Planetary associations according to Schlesinger			Greek Apollonian: structure and scale descending. See Table 10. Planetary associations according to Nicomachos and Rudolf Steiner's indication to Lewerenz			Church modes: structure Apollonian. Scales descending and ascending. See Table 10		
1st Tetrachord	D.I.	2nd Tetrachord	1st Tetrachord	D.I.	2nd Tetrachord	1st Tetrachord	D.I.	2nd Tetrachord
1) Hypodorian-Saturn 1/16 1/15 1/13 1/12 16:15 15:13 13:12	12:11	1/11 1/10 1/9 1/8 11:10 10:9 9:8	Hypodorian-Saturn a g f e 9:8 9:8 256:243	9:8	d c b a 9:8 256:243 9:8	Aeolian or Hypodorian a b c d 9:8 256:243 9:8	9:8	e f g a 256:243 9:8 9:8
2) Hypophrygian-Jupiter 1/181/16 1/15 1/13 9:8 16:15 15:13	13:12	1/12 1/11 1/10 1/9 12:11 11:10 10:9	Hypophrygian-Jupiter g f e d 9:8 256:243 9:8	9:8	c b a g 256:243 9:8 9:8	Hypophrygian or Lokrian b c d e 256:243 9:8 9:8	256:243	f g a b 9:8 9:8 9:8
3) Hypolydian-Mars 1/20 1/18 1/16 1/15 10:9 9:8 16:15 1/16 1/14 8:7	15:13 14:13	1/13 1/12 1/11 1/10 13:12 12:11 11:10	Hypolydian-Mars f e d c 256:243 9:8 9:8	256:243	b a g f 9:8 9:8 9:8	Ionian or Hypolydian c d e f 9:8 9:8 256:243	9:8	g a b c 9:8 9:8 256:243
4) Dorian-Sun 1/22 1/20 1/18 1/16 11:10 10:9 9:8	8:7 16:15	1/14 1/13 1/12 1/11 14:13 13:12 12:11 1/15 1/13 15:13	Dorian-Sun e d c b 9:8 9:8 256:243	9:8	a g f e 9:8 9:8 256:243	Dorian or Hypomixolydian d e f g 9:8 256:243 9:8	9:8	a b c d 9:8 256:243 9:8
5) Phrygian-Venus 1/24 1/22 1/20 1/18 12:11 11:10 10:9	9:8	1/16 1/15 1/13 1/12 16:15 15:13 13:12 1/16 1/14 1/13 8:7 14:13	Phrygian-Venus d c b a 9:8 256:243 9:8	9:8	g f e d 9:8 256:243 9:8	Phrygian e f g a 256:243 9:8 9:8	9:8	b c d e 256:243 9:8 9:8
6) Lydian-Mercury 1/26 1/24 1/22 1/20 13:12 12:11 11:10	10:09	1/18 1/16 1/15 1/13 9:8 16:15 15:13 1/16 1/14 1/13 8:7 14:13	Lydian-Mercury c b a g 256:243 9:8 9:8	9:8	f e d c 256:243 9:8 9:8	Lydian f g a b 9:8 9:8 9:8	256:243	c d e f 9:8 9:8 256:243
7) Mixolydian-Moon 1/28 1/26 1/24 1/22 14:13 13:12 12:11	11:10	1/20 1/18 1/16 1/14 10:9 9:8 8:7	Mixolydian-Moon b a g f 9:8 9:8 9:8	256:243	e d c b 9:8 9:8 256:243	Mixolydian g a b c 9:8 9:8 256:243	9:8	d e f g 9:8 256:243 9:8

D.I. = dividing interval between the tetrachords

12 Diatonic Scales Based on the True-tone Row and the Creation of the World's Soul According to Plato

In chapter 6, we saw how Pythagoras created the diatonic Dorian true-tone octachord from the old Terpander heptachord and that this became the central scale of the Greek Apollonian music system. When the interval sequence of this scale is expressed numerically, it shows an amazing similarity to the third part of the 'creation of the world's soul' in Plato's *Timaeus*.

An in-depth examination of three pertinent paragraphs in the *Timaeus* will be attempted to work out the form principles of the diatonic true-tone scales (church modes). The tetrachords of these scales are given in Table 8.

The three paragraphs read:

1) First He [the Creator] took one portion from the whole;
Then He took a portion double of this;
then a third portion, half as much again as the second portion, that is, three times as much as the first;
the fourth portion He took was twice as much as the second;
the fifth three times as much as the third;
the sixth eight times as much as the first; and
the seventh twenty-seven times as much as the first.*

2) After that He went on to fill up the intervals in the series of the powers of 2 and the intervals in the series of powers of 3 in the following manner:
He cut off yet further portions of the original mixture, and set them in between the portions above rehearsed, so as to place two means in each interval—one a mean which exceeded its extremes and was by them exceeded by the same proportional part or fraction of each of the extremes respectively;† the other a mean which exceeded one extreme by the same number or integer as it was exceeded by its other extreme.‡

3) 'And whereas the insertion of these links formed fresh intervals in the former intervals, that is to say, intervals of 3:2 [$1\frac{1}{2}$] and 4:3 [$1\frac{1}{3}$] and 9:8 [$1\frac{1}{8}$], He went on to fill up the 4:3 intervals with 9:8 intervals. This still left over in each case a fraction, which is represented by the terms of the numerical ratio 256:243.

Quoted from: The Loeb Classical Library *Plato*, vol. VII (including the notes).

Whether we are concerned with string-length division or frequencies, tones (Plato = numbers) and intervals (Plato = fractions) need to be clearly distinguished from one another. It must also be remembered that two tones are needed for an interval.

---

* These seven numbers may be arranged in two branches to show the two series of which the *Timaeus* immediately goes on to speak:

1 (the 1st)
2 (the second) 3 (the third)
4 (the fourth) 9 (the fifth)
8 (the sixth) 27 (the seventh)

The left-hand branch contains the 'double intervals', i.e., powers of 2: the right-hand one the 'triple intervals', i.e., powers of 3.

† The harmonic mean.

‡ The arithmetical mean.

The Neo-Pythagoreans, for example Nicomachos of Gerasa,[4] calculated the numerical row 9:8, 9:8, 256:243, 9:8, 9:8, 9:8, 256:243 from the above three paragraphs and showed how the Pythagorean octachord arises in the doubled interval (octave) referred to in the third paragraph. In the twentieth century, the mathematician Ernst Bindel also took up these calculations.[11a]

Nicomachos' and Bindel's method of calculation will hardly give the form principles for all seven diatonic true-tone scales. Especially in the second paragraph, Plato mentions not just a twofold but also a threefold interval row. An explanation of the threefold row poses problems for most commentators of Plato because they only place two fifths separated by a fourth into the 3:1 interval of the twelfth. Here, the aim is to extend the method so that the form principles for the diatonic true-tone scales will emerge quite naturally.

Apart from full acknowledgement of the true-tone row, there is a third element to be considered. If we relate Plato's numbers to music, both directions of movement operative in the Apollonian stream since the turning-point of time need to be taken into account—the pre-Christian descending direction of the undertone-row degrees (string-length division), and the Christian ascending direction of the overtone-row degrees (frequencies).*

If we do this and replace the original numbers with the corresponding tones, we get all seven tones of the diatonic Apollonian true-tone scale, as well as their form principles, as shown in the two proceeding tables.

The first paragraph contains the portion or number row 1, 2, 3, 4, 8, 9, 27. If one wishes to remain in the realm of non-altered tones, then there are only two possibilities for the initial tone '1'—in string-length division e^4, and in frequencies ${}_1C$. All other initial tones would result in altered tones. Apart from this, these seven numbers, or portions, only give four different tones, as numbers 2, 4 and 8 are octave tones of 1. In string-length division these would be tones $\underline{e}^4 = 1$ ($e^3 = 2$, $e^2 = 4$, $e^1 = 8$), $\underline{a}^2 = 3$, $\underline{d}^1 = 9$ and $\underline{G} = 27$. The four underlined tones are in descending proportions of a 3:1 twelfth to each other. As fifths and fourths in the corresponding octave, these give the tones of the open strings of the violin and the double bass. In frequencies these would be tones ${}_1\underline{C} = 1$ ($C = 2$, $c = 4$, $c^1 = 8$), $\underline{G} = 3$, $\underline{d}^1 = 9$, $\underline{a}^2 = 27$, the underlined tones are in the ascending proportion of 3:1 which, when brought into the corresponding octaves, can be played on the open strings of the viola and cello. Together, the tones of both rows give the row e^4, a^2, d^1, G and ${}_1C$, each tone being a twelfth apart.

When Plato's numbers are taken as interval rows, the following intervals arise:

String-length division-row, descending (read from right to left):

Degree:	27	:	8	:	9	:	4	:	3	:	2	:	1
Interval:		octave + major 6th		whole tone		9th		4th		5th		octave	

Frequencies-row, ascending (read from left to right):

Degree:	1	:	2	:	3	:	4	:	9	:	8	:	27
Interval:		octave		5th		4th		9th		whole tone		octave + major 6th	

In the second paragraph, Plato divides the portions of the first paragraph into a twofold and a threefold interval row. Both rows include the geometric mean twice: in the octave row as 4:2:1 and 8:4:2, and in the twelfth row as 9:3:1 and 27:9:3. The geometric mean is thus an organic part of these rows from the outset. Plato then places the harmonic mean and arithmetic mean in these two rows (see Table 9, lines 1, 2, 6, 7). In the octave row, the harmonic mean gives in string-length division the tone true B and in frequencies the tone true F:

* See chapter 3 and Bibliographic References 26 and 62.

Read from right to left:

		harmonic mean		harmonic mean		harmonic mean	
Degrees in string-length division:	8	$5\frac{1}{3}$	4	$2\frac{2}{3}$	2	$1\frac{1}{3}$	1
Tones	true e^1	true b^1	true e^2	true b^2	true e^3	true b^3	true e^4

Read from left to right:

		harmonic mean		harmonic mean		harmonic mean	
Degrees in frequencies:	1	$1\frac{1}{3}$	2	$2\frac{2}{3}$	4	$5\frac{1}{3}$	8
Tones	true $_1C$	true $_1F$	true C	true F	true c	true f	true c^1

The arithmetic mean of the twofold row gives new numbers but these are in a proportion of 2:1 to the number three which is contained in the threefold row. Neither string-length division nor frequencies therefore give new tones. But together the harmonic mean and arithmetic mean place intervals into the twofold interval row that were previously only given in the first paragraph. They divide the octave into two equally large 4:3 fourths divided by a 9:8 major second in the middle (see Table 9, lines 2 and 8).

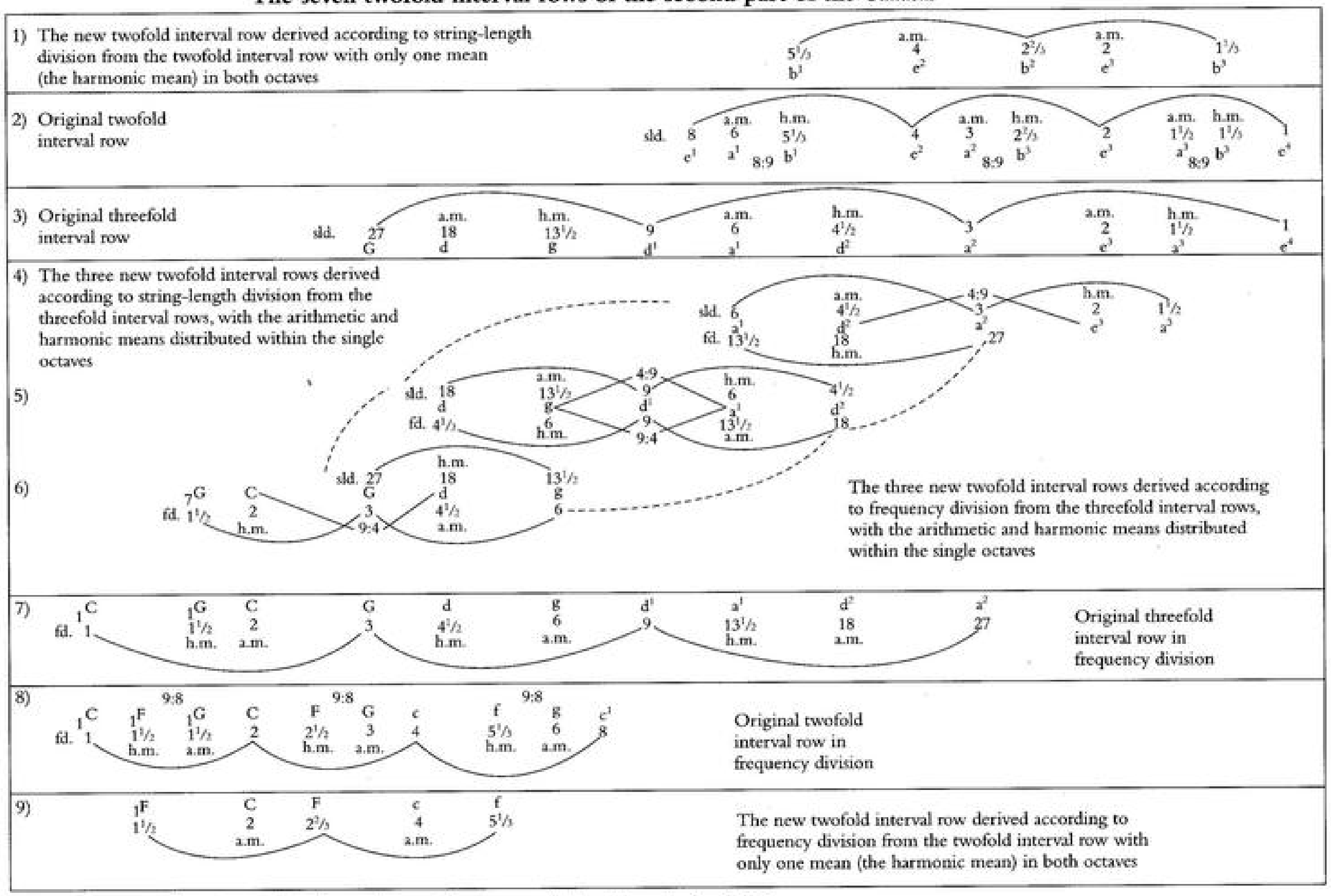
In the threefold row, which contains the numbers 1, 3, 9 and 27, string-length division gives tones true e^4, true a^2, true d^1, true G and frequencies the tones true $_1C$, true G, true d^1, true a^2. Division of this row by the harmonic mean and arithmetic mean gives octave intervals to the tones true a^2, true d^1 and true G (see lines 4, 5 and 6 in Table 9). The tones of the double octave $1\frac{1}{2}$:6 are, in string-length division, octaves of true a^2, and in frequencies, octaves of true G. The true D tones of the middle double octave $4\frac{1}{2}$:18 belong to both string-length division and frequencies, so that only three and not four double octave rows are produced by this division. Apart from this, the above division also brings about that each double octave derived in this way contains either (but not both) the harmonic mean or the arithmetic mean. This produces the 9:4 ninth intervals, which we may also call octave-exceeding major seconds.[30]

The important point (not so far taken into account by Plato commentators) that the three new double octaves are twofold intervals which contain the arithmetic mean and harmonic mean needs to be stressed. To place the harmonic mean and arithmetic mean into each octave of these new double octaves, which would be in keeping with Plato, the missing means need to be carried over from the single octaves. Fifths, fourths and major seconds are then produced in the same way as they were in the original twofold intervals.

Two further twofold interval rows can be derived from the original twofold row (lines 1 and 9 in Table 9). The first is double octaves of the harmonic mean, the second, double octaves of the arithmetic mean. The double octaves of the arithmetic mean are already contained in the new row of the double octaves in the threefold interval row (lines 4 and 6 in Table 9). New tones will therefore only arise from the harmonic mean (Table 9, lines 1 and 9). Under the conditions shown in Table 9, these new double octave rows have the characteristic that their two single octaves contain not two, but only one mean, namely, the arithmetic mean: 2 between $1\frac{1}{3}$ and $2\frac{2}{3}$, and 4 between $2\frac{2}{3}$ and $5\frac{1}{3}$. This one mean divides each octave in this octave row into a perfect fourth and a perfect fifth. But as these octaves only contain one of the means, there is no mean dividing interval, i.e., no major second can be produced. The divisions are therefore incomplete.

Seven different twofold interval rows thus arise from the second paragraph—the two original (Table 9 and Table 10, lines 2 and 8) and the five derived rows (Table 9 and Table 10, rows 1, 4, 5, 6 and 9). These seven twofold interval rows establish the parameters and basic structure for the seven diatonic true-tone scales of the Greek Apollonian musical stream and the church modes. Our analysis of paragraph 2 clearly shows, however, that only two scales are founded on the original twofold interval row—the Greek true Dorian octachord and its inversion, the church Ionian true-tone C major scale. The other five owe

Table 9
The seven twofold interval rows of the second part of the *Timaeus*



Abbreviations: a.m. = arithmetic mean; h.m. = harmonic mean; fd. = frequency division; sld. = string-length division.

their origin to the 'derived' twofold interval row. This enables us to understand the central position of the Dorian Sun scale in Greek music.

Only five of the seven twofold interval rows gained in this way are complete—the two original rows and the three rows derived from the threefold interval rows (Table 9, lines 2, 4, 5, 6 and 8). The other two are incomplete, as we have seen (Table 9, lines 1 and 9). The five major seconds contained in the Apollonian diatonic true-tone scales are created in the five complete interval rows (Table 10, lines 2, 4, 5, 6 and 8). On the basis of the second paragraph we may therefore say that the form principles that lie at the basis of these diatonic true-tone scales are the harmonic mean and arithmetic mean between five true tones and their octaves, namely, true $_1$C, true G, true d^1, true a^2, true e^4. Lying next to each other (Table 9), these form a twelfth row.

The third paragraph tells us how, through the hand of the Divine Creator, all 4:3 fourth intervals are filled in with 9:8 seconds, with a remainder with the proportion 256:243 (90.225 cents) left over in each 4:3 fourth interval. This remainder is called limma. In other words, we are told how the seven twofold interval rows, with the five major seconds derived through the above-mentioned form principles, are filled in. This is how the five complete tetrachords and one tetrachord of each of the two incomplete Apollonian true-tone scales arise. Three of the complete scales contain two of the same tetrachords. The one descending from e to E contains two Dorian tetrachords, the one descending from d to D, two Phrygian tetrachords, and the one descending from c to C, two Lydian tetrachords. In accordance with their tetrachord names, these three scales are called Dorian, Phrygian and Lydian. They arise from the two original interval rows and the middle one of the three complete 'derived' twofold interval rows. The other two complete scales each contain two different tetrachords. The descending Hypodorian scale from a to A contains one Hypodorian and one Dorian tetrachord. The descending Hypophrygian scale from g to G contains one Phrygian and one Lydian tetrachord. They arise from the two other complete twofold interval rows derived from the threefold interval row. In contrast to this, the two incomplete scales have only one perfect fourth tetrachord—the Mixolydian scale descending from b to B has a Dorian tetrachord as its second tetrachord, and the Hypolydian scale, descending from f to F, has a Lydian tetrachord as first tetrachord. In order to complete them as octave scales, using the five mentioned major seconds, the two tetrachords must be separated by the limma 256:243. The other irregular tetrachord not contained by a mean division is enlarged by 113,685 cents and thereby contained within an augmented fourth (see Table 10, line 12, and Table 8). From this point of view all seven diatonic true-tone scales can be derived from the third paragraph.

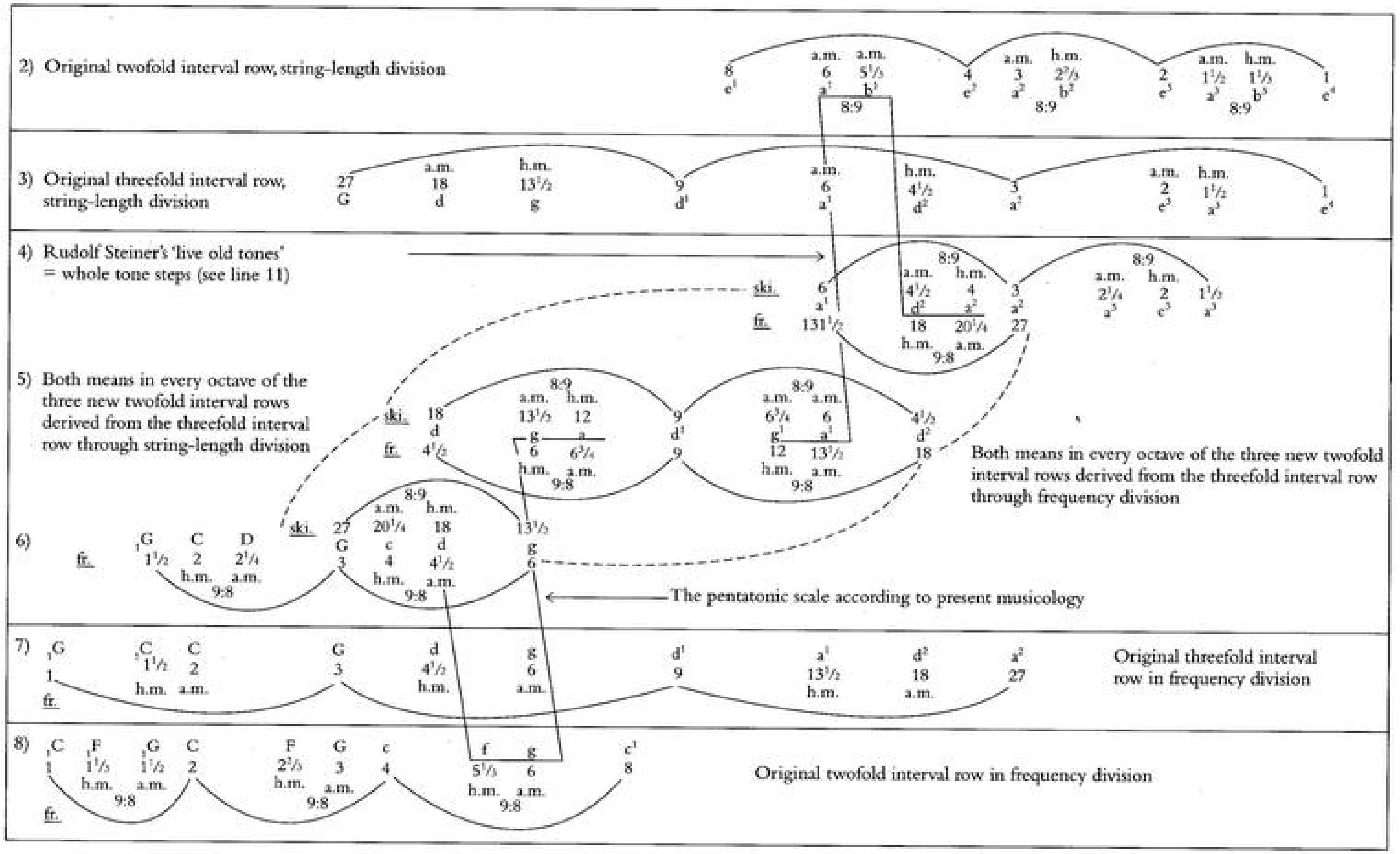
As already mentioned, the classical viewpoint is narrower.[4, 11a] With this, the fourths of the original twofold interval row of paragraph three are only filled out with 9:8 major seconds and given the number-row 9:8, 9:8, 256:243. This gives the Apollonian Dorian tetrachord, and, as a scale, the true Dorian octachord, the Apollonian Sun scale (see chapter 19). Table 9 shows how this central scale easily becomes the true-tone C major scale when inverted. If one combines the classical viewpoint with the one developed here it can be said that the form principles of the true-tone scales show a close relationship for all seven scales to the creation of the world's soul as given by Plato in his *Timaeus*; the classical viewpoint does, however, only consider the central Sun scale.

For the sake of completeness, it still needs to be mentioned that the seven true tones, which form the five major seconds, also produce two pentatonic scales. The frequencies row, c, d, f, g, a, begins upon c = 4, which is frequently considered by present musicologists to be the original form of the pentatonic scale.[13, 14, 16, 34, 38] The string-length division row e, d, b, a, g or d, e, g, a, b begins upon $e^2 = 4$. Rudolf Steiner called the latter 'the five ancient tones'.[19, 30] Both scales are indicated by the connecting lines in Table 10 (lines 6, 5, 8 and 4, 2, 5).

It is worth noting that only octaves, fifths, fourths and major seconds created by means of the named form principles are those which the human ear once again, and without exception, experiences to be genuine. It follows from this that intervals genuine to the ear, if added together or subtracted from one another, produce other intervals that are equally genuine to the ear.

The next chapter will show that it is possible for the human ear to find a chromatic scale equal to modern musical demands that is actuated by the three dividing principles given in the *Timaeus*—the harmonic mean, arithmetic mean and geometric mean. This is based on the true-tone C major scale. In contrast to the twelve-tone equal-tempered scale, this new chromatic scale is made up entirely of intervals that sound genuine to the ear and can thus lead to a deepened experience of chromaticism.

Table 10
The form principles of the diatomic true-tone scales according to the *Timaeus*



10) *Timaeus* paragraph 2 (lines, 2, 4, 5, 6, and 8): The five whole tones of the twofold interval row:

$_1F : {_1G}$ C : D   F : G   c : d   f : g : a $g^1 : a^1 : b^1$ $d^2 : e^2$ $a^2 : b^2$ $d^3 : e^3$ $a^3 : b^3$

11) The pentatonic scale according to present musicology[28] (lines 6, 8 and 5):   The five old tones according to Rudolf Steiner[29] (lines 5, 2 and 4):

c d f g a / d^1 e^1 g^1 a^1 b^1

12) *Timaeus* paragraph 3: All 4:3 (fourth) intervals filled in with 9:8 whole tones give the scale:

$_1C$ $_1D$ $_1E$ $_1F$ $_1G$ $_1A$ $_1B$ C D E F G A B c d e f g a b c^1 d^1 e^1 f^1 g^1 a^1 b^1 c^2 d^2 e^2 f^2 g^2 a^2 b^2 c^3 d^3 e^3 f^3 g^3 a^3 b^3 c^4 d^4 e^4

This scale contains:

fr .= ascending fifth-tones scales‡ to be read from left to right →

ski.= descending fifth-tones scales† to be read from right to left ←

Pythagorian Dorian octachord called Phrygian in the church modes
e^1 f^1 g^1 a^1 b^1 c^2 d^2 e^2 f^2 g^2 a^2 b^2 c^3 d^3 e^3 f^3 g^3 a^3 c^4 d^4 e^4

church Dorian
D E F G A B c d
= Greek Phrygian, church Dorian
d^1 e^1 f^1 g^1 a^1 b^1 c^2 d^2 e^2 f^2 g^2 a^2 b^2 c^3 d^3

Christian Dorian = Major§
$_1C$ $_1D$ $_1E$ $_1F$ $_1G$ $_1A$ $_1H$ C D E F G A B c
= Greek Lydian, church Hypomyxolydian or Ionic
c^1 d^1 e^1 f^1 g^1 a^1 b^1 c^2 d^2 e^2 f^2 g^2 a^2 b^2 c^3

Greek Hypolydian, church Lydian
f^1 g^1 a^1 b^1 c^2 d^2 e^2 f^2

Greek Hypophrygian, church Mixolydian
g^1 a^1 b^1 c^2 d^2 f^2 g^2

Christian Dorian = Minor
$_1A$ $_1H$ C D E F G A
Greek Hypodorian, church Hypodorian or Aeolian
a^1 b^1 c^2 d^2 e^2 f^2 g^2 a^2

Greek Mixolydian, church Hypophrygian
b^1 c^2 d^2 e^2 f^2 g^2 a^2 b^2

Abbreviations: sld. = string-length division; fr. = frequency division; h.m. = harmonic mean; a.m. = arithmetic mean

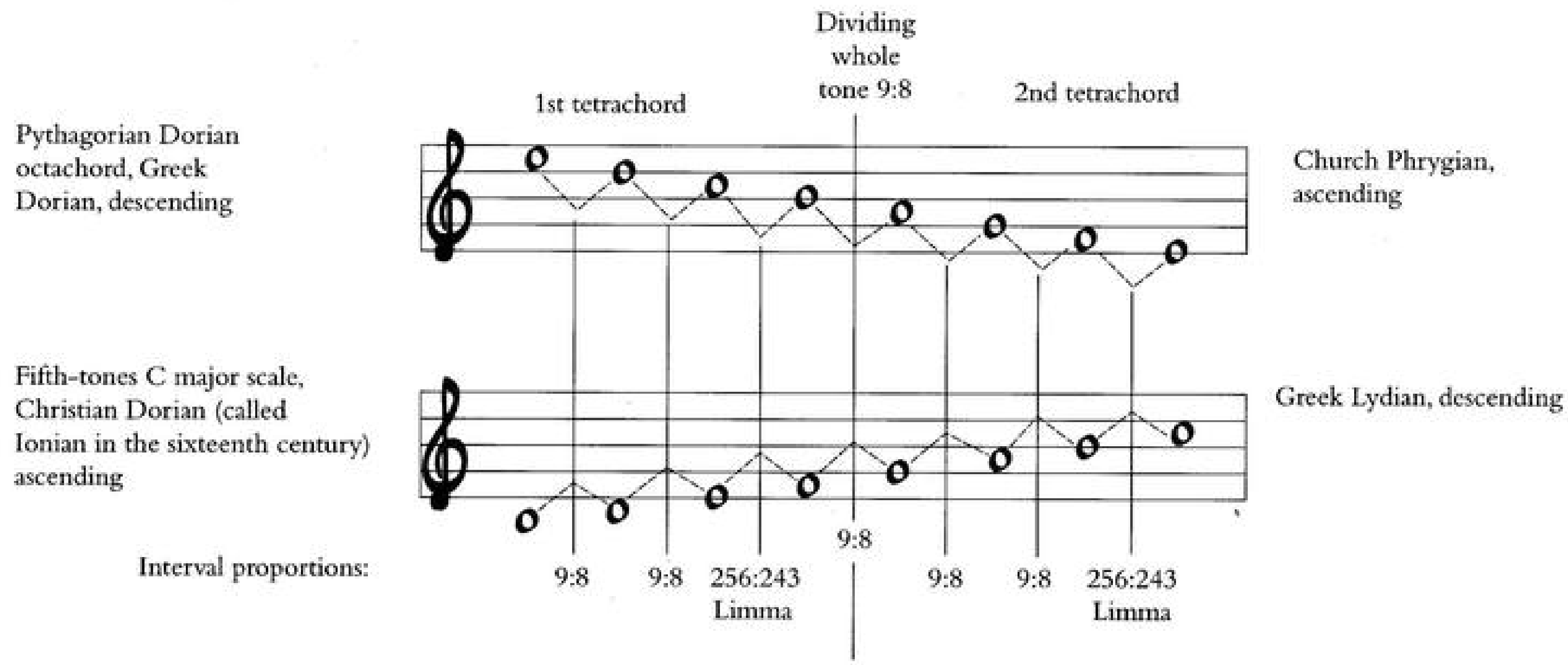
* For clarity's sake, in Tables 9, 10 and 12 the corresponding lines have the same number. In Table 10 the lines 1 and 9 have been left out as they do not belong to the form principles of the diatonic fifth-tone scale.

† The bows indicate the true semitone 256:243 = Limma.

‡ See Table 8 for the tetrachord structure of these scales.

§ This scale has been called the Christian Dorian major scale because it represents an ascending inversion of the Pythagorean Dorian octachord (see Table 11).

Table 11
Comparison between the tetrachord divisions of the true Dorian octachord and the true C major scale



13 The Scale of Twelve Fifths

In 1962, departing from the true-tone C major scale presented in the last chapter, the author aurally found a twelve-toned chromatic scale in which free passage through all major and minor keys is possible and which is made up entirely of aurally genuine non-tempered intervals. This scale is called the scale of twelve fifths[12] because it is made up of two groups of true tones originating in the twelfth row. The first group is made up of the seven tones F, C, G, D, A, E, B from the true Dorian octachord and the true-tone C major scale. The second group progresses in a row of twelfths from the geometric mean of the octave c = 128 Hz: c^1 = 256 Hz to create five new tones. The geometric mean of the octave C–c is neither f♯ nor g♭ but the tonal centre between these two tones.* In order to make clear that they are geometric mean true tones, and to differentiate them from ordinary and equal-tempered tones, they are called 'Gelis', 'Delis', 'Alis', 'Elis' and 'Belis'.†

It is important to stress that the five geometric mean tones were found by ear. The qualities of these new intervals differ from that of the just and true intervals and are therefore called 'formed' intervals. These formed intervals harmonize with the true intervals to make well-sounding harmonies and characteristic dissonances. Together, both rows of tones form a genuine sounding chromatic scale and 24 equally genuine-sounding major and minor scales that are suitable for the realization of all works of music on instruments of fixed tuning. A trial with any instrument tuned in this way will show this to be the case, making the unsatisfying, false-sounding equal-tempered tuning unnecessary.‡

Several years after publishing this new tuning method in *Das Goetheanum*,[12] the author chanced on the books of Barbour[39] and Yasser[34] in the New York Public Library. Both proved to be very interesting. Barbour mentions how, as early as 1518, Heinrich Schreyber (who also called himself Henricus Grammateus) recommended a chromatic twelve-toned scale which he had gained by geometric mean division.[40] Barbour only calculated the monochord divisions of Grammateus, however, and therefore described this division as being one amongst many tempered tunings. If he had made the admittedly major effort to produce these tones on a monochord as the author did, he would have heard immediately that these do not produce a 'tempered' scale. If Grammateus' indications are followed exactly, the result is an aurally genuine scale which is identical to the scale of twelve fifths.

Grammateus' book[40] (written in old-style German that is difficult to understand) deals mainly with calculation and mathematics and only includes a short chapter on music with the above-mentioned monochord division. Grammateus, basing himself on Pythagoras, first described the major scale of the seven perfect fifths and all the interval proportions belonging to it. He shows how the limma (256:243) is left over when the ditonus (81:64) is 'removed' from the fourth (4:3). He then described how the apotomé (2187:2048) comes about when a major second (9:8) is made smaller by a limma 256:243.[40] According to Ellis[14] these interval proportions can be converted to the cent values which are in common use today. A fourth of 498.045 cents, less a ditonus of 407.820 cents, gives a limma of 90.225 cents; and a major second of 203.910 cents, less a limma of 90.225 cents, gives an apotomé of 113.685 cents.

Grammateus went on to explain how a monochord board was to be divided in order to gain all seven of the tetrachords in use at the time. There follows a description of how he divided each major second, marked on the monochord board, in order to get, in his own words, a 'semitonium minor' (limma) and a 'semitonium major' (apotomé). This text was apparently not taken into account by Barbour. If one

* This scale is not to be confused with the fictitious tone row of tempered tuning shown in Table 7 as a mathematical row of $\sqrt[12]{2}$ numbers. In the scale of twelve fifths the geometric mean of the octave which creates the aurally genuine equal-tempered tritone takes on a central role.

† See footnote* on page 8 (chapter 2).

‡ Many musicians have testified that, without exception, all intervals sound genuine.

Table 12

The form principles of the scale of twelve fifths-tones scale. The harmonic, geometric and arithmetic means placed in the fifth 'twofold interval spaces' of the named octave rows of the *Timaeus*

2)

4)

5)

		a.m.	g.m.	h.m.
sld.:	18	$13^1/_2$	12.728	12
	d	g	alis	a
fr.:	$4^1/_2$	6	6.364	$6^3/_4$
		h.m.	g.m.	a.m.

6)

		a.m.	g.m.	h.m.		a.m.	g.m.	h.m.	
sld.:	54	$40^1/_2$	38.184	36	27	$20^1/_4$	19.092	18	$13^1/_2$
	,G	C	Delis	D	G	c	delis	d	g
fr.:	$1^1/_2$	2	2.1213	$2^1/_4$	3	4	4.243	$4^1/_2$	6
		h.m.	g.m.	a.m.		h.m.	g.m.	a.m.	

8)

		a.m.	g.m.	h.m.		a.m.	g.m.	h.m.		a.m.	g.m.	h.m.	
sld.:	81	$60^3/_4$	57.2756	54	$40^1/_2$	$30^3/_8$	28.6378	27	$20^1/_4$	15.1875	14.3189	$13^1/_2$	$10^1/_8$
	,C	,F	1Gelis	,G	C	F	Gelis	G	c	f	gelis	g	c^1
fr.:	1	$1^1/_3$	1.4142	$1^1/_2$	2	$2^2/_3$	2.8284	3	4	$5^1/_3$	5.659	6	8
		h.m.	g.m.	a.m.		h.m.	g.m.	a.m.		h.m.	g.m.	a.m.	

10) The five whole tones and their geometric mean tones of the twofold interval row (the sum of lines 2, 4, 5, 6 and 8):

,F 1Gelis ,G — C Delis D — F Gelis G — c delis d — f gelis g alis a

1Gelis — Delis — Gelis — delis — gelis — alis

11)

The newly gained pentatonic scale[30] (lines 2, 4, 5, 6 and 8): delis — gelis — alis

Pentatonic scale according to modern musicology[28] (lines 6, 8 and 5): c — d — f — g — a

12) All 4:3 fourth intervals filled in with 9:8 whole tones, 67.882:64 semitones and 256:243 Limmas result in the twelve fifth tones scale:

,C 1Delis ,D 1Elis ,E ,F 1Gelis ,G 1Alis ,A 1Belis ,B C Delis D Elis E F Gelis G Alis A Belis B c delis d elis e f gelis g alis a belis b c

Abbreviations: sld. = string-length division; fr. = frequency division; g.m. = geometric mean; h.m. = harmonic mean; a.m. = arithmetic mean

*For clarity's sake in Tables 9 and 12 the corresponding lines have the same numbers

etic means

a.m. g.m. h.m. a.m. g.m. h.m. a.m. g.m. h.m.
6 5.6568 5 1/3 4 3 2.8284 2 2/3 2 1 1/2 1.4142 1 1/3 1
a¹ belis¹ b¹ e² a² belis² b² e³ a³ belis³ b³ e⁴
13 1/2 14.3189 15.1875 20 1/4 27 28.6378 30 3/8 40 1/2 54 57.2756 60 3/4 81
h.m. g.m. a.m. h.m. g.m. a.m. h.m. g.m. a.m.

a.m. g.m. h.m. a.m. g.m. h.m.
sld.: 6 4 1/2 4.2427 4 3 2 1/4 2.1213 2 1 1/2
a¹ d² elis² e² a² d³ elis³ e³ a³
fr.: 13 1/2 18 19.092 20 1/4 27 36 38.184 40 1/2 54
h.m. g.m. a.m. h.m. g.m. a.m.

a.m. g.m. h.m. | a.m. g.m. h.m.
13 1/2 12.728 12 | 6 1/4 6.364 6 4 1/2
g alis | g¹ alis¹ a¹ d²
6 6.364 6 1/4 | 12 12.728 13 1/2 18
h.m. g.m. a.m. | h.m. g.m. a.m.

13 1/2
g
6

m. g.m. h.m.
875 14.3189 13 1/2
gelis g
/3 5.659 6
m. g.m. a.m.

8):
gelis g alis a | g¹ alis¹ a¹ belis¹ b¹ d² elis² e² a² belis² b² d³ elis³ e³ a³ belis³ b³
gelis alis | alis¹ elis² belis² elis³

gelis alis | alis¹ belis¹ elis²
f g a | e g¹ a¹ b¹ The five ancient tones (= 3 whole tone steps)[20] according to Rudolf Steiner (lines 5, 2 and 4)

elve fifth tones scale:
f gelis g alis a belis b | e f gelis¹ g¹ alis¹ a¹ belis¹ b¹ c² delis² d² elis² e² f² gelis² g² alis² a² belis² b² c³ delis³ d³ elis³ e³ f³ gelis³ g³ alis³ a³ belis³ b³ c⁴ delis⁴ d⁴ elis⁴ e⁴

mean

Table 12

The form principles of the scale of twelve fifths-tones scale. The harmonic, geometric and arithmetic means placed in the fifth 'twofold interval spaces' of the named octave rows of the *Timaeus*

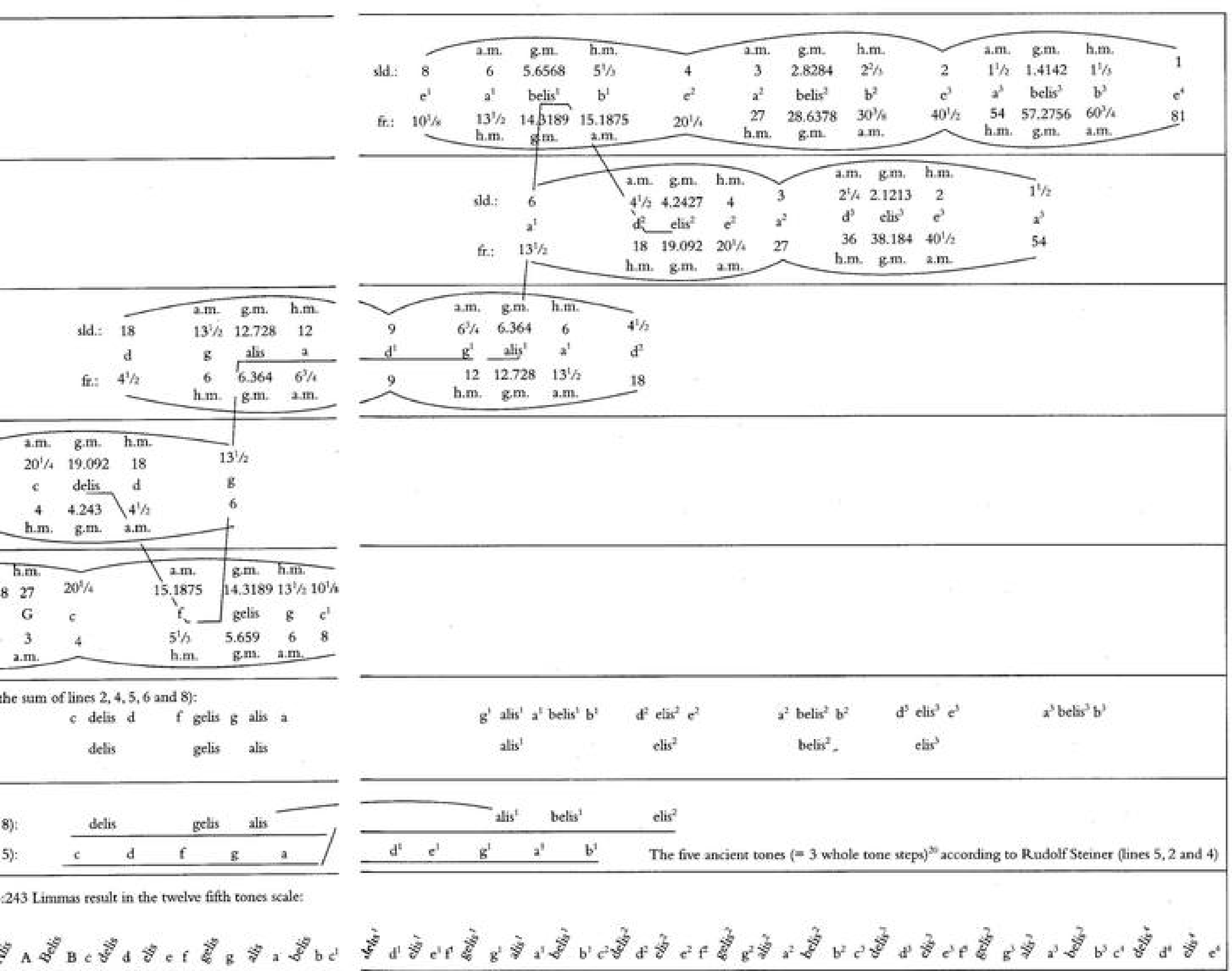
2)		a.m. g.m. h.m. sld.: 8 6 5.6568 $5^1/_3$ 4 e^1 a^1 $belis^1$ b^1 e^2 fr.: $10^1/_8$ $13^1/_2$ 14.3189 15.1875 $20^1/_4$ h.m. g.m. a.m.
4)		a.m. g.m. sld.: 6 $4^1/_2$ 4.242 a^1 d^2 $elis^2$ fr.: $13^1/_2$ 18 19.09 h.m. g.m.
5)	a.m. g.m. h.m. sld.: 18 $13^1/_2$ 12.728 12 d g alis a fr.: $4^1/_2$ 6 6.364 $6^3/_4$ h.m. g.m. a.m.	a.m. g.m. h.m. 9 $6^3/_4$ 6.364 6 $4^1/_2$ d^1 g^1 $alis^1$ a^1 d^2 9 12 12.728 $13^1/_2$ 18 h.m. g.m. a.m.
6)	a.m. g.m. h.m. a.m. g.m. h.m. sld.: 54 $40^1/_2$ 38.184 36 27 $20^1/_4$ 19.092 18 $13^1/_2$ $_1$G C Delis D G c delis d g fr.: $1^1/_2$ 2 2.1213 $2^1/_4$ 3 4 4.243 $4^1/_2$ 6 h.m. g.m. a.m. h.m. g.m. a.m.	
8)	a.m. g.m. h.m. a.m. g.m. h.m. a.m. g.m. h.m. sld.: 81 $60^3/_4$ 57.2756 54 $40^1/_2$ $30^3/_8$ 28.6378 27 $20^1/_4$ 15.1875 14.3189 $13^1/_2$ $10^1/_8$ $_1$C $_1$F 1Gelis $_1$G C F Gelis G c f gelis g c^1 fr.: 1 $1^1/_3$ 1.4142 $1^1/_2$ 2 $2^2/_3$ 2.8284 3 4 $5^1/_3$ 5.659 6 8 h.m. g.m. a.m. h.m. g.m. a.m. h.m. g.m. a.m.	
10)	The five whole tones and their geometric mean tones of the twofold interval row (the sum of lines 2, 4, 5, 6 and 8): $_1$F 1Gelis $_1$G C Delis D F Gelis G c delis d f gelis g alis a 1Gelis Delis Gelis delis gelis alis	g^1 $alis^1$ a^1 $belis^1$ b^1 d^2 $elis^2$ $alis^1$ $elis^2$
11)	The newly gained pentatonic scale[30] (lines 2, 4, 5, 6 and 8): delis gelis alis Pentatonic scale according to modern musicology[28] (lines 6, 8 and 5): c d f g a	$alis^1$ $belis^1$ $elis^2$ d^1 e^1 g^1 a^1 b^1 The fi…
12)	All 4:3 fourth intervals filled in with 9:8 whole tones, 67.882:64 semitones and 256:243 Limmas result in the twelve fifth tones scale: $_1$C 1Delis $_1$D 1Elis $_1$E $_1$F 1Gelis $_1$G 1Alis $_1$A 1Belis $_1$B C Delis D Elis E F Gelis G Alis A Belis B c delis d elis e f gelis g alis a belis b c^1	[illegible] d^1 $elis^1$ e^1 f^1 $gelis^1$ g^1 $alis^1$ a^1 $belis^1$ b^1 c^2 $delis^2$ d^2 $elis^2$ e^2

Abbreviations: sld. = string-length division; fr. = frequency division; g.m. = geometric mean; h.m. = harmonic mean; a.m. = arithmetic mean

*For clarity's sake in Tables 9 and 12 the corresponding lines have the same numbers

eometric and arithmetic means

Timaeus



harmonic mean; a.m. = arithmetic mean

actually compares the geometric monochord division of Grammateus with his interval proportions, it is evident that they do not agree with one another at all. The drawn division of the monochord for the major second actually gives the major second's geometric mean of 101.955 cents. The geometric monochord division of Grammateus therefore resulted in the scale of twelve fifths even though he himself did not appear to be aware of this.

In his *A Theory of Evolving Tonality*,[34] Joseph Yasser described how the twelve-tone chromatic just and equal-tempered scales have arisen from a fusion of the diatonic and pentatonic scales. According to this original and noteworthy conception, the pentatonic part gives rise to the five semitone steps, which, together with the seven-degree diatonic scale, give the present twelve-tone scale. Analytically this view proves to be correct for all twelve-tone chromatic scales.

The scale of twelve fifths also corresponds with Yasser's requirements. It contains seven diatonic tones that form the scale c, d, e, f, g, a, b, (c) and five twelfth-tones that make up the pentatonic scale delis, elis, gelis, alis, belis (Table 12 on pp. 60 and 61, line 11). But, as it is the quality of the intervals and tones which gives beauty and satisfaction to music, the aural genuineness of the tones and intervals is at least as important as the formal structure.

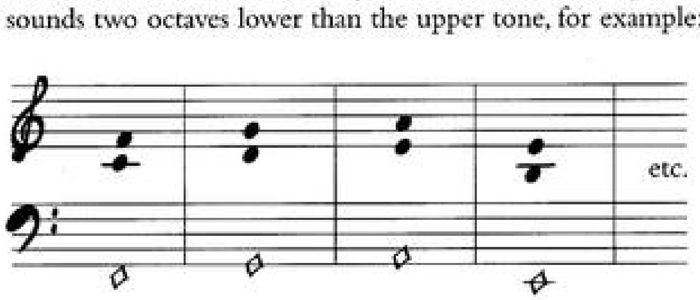
The genuineness of the diatonic tones, created with the harmonic and arithmetic means, has already been shown in chapter 12. What is the situation with the formed intervals which transform the diatonic into a chromatic scale? As we have seen, these formed intervals are built through the geometric means of the octaves of the tones of the twelfth-row $_1C$, G, d^1, a^2, e^4 (see Table 12). As the geometric mean also belongs to the basic laws of music, the aurally experienced genuineness of the formed intervals is substantiated, and the scale of twelve fifths proves itself to be a logical further development of the true-tone C major scale.

Notes on tuning an instrument to the scale of twelve fifths*

Experience has shown that it is advisable to use three tuning forks tuned exactly to c^1 = 256.000 Hz, $gelis^1$ = 362.04 Hz and a^1 = 432.000 Hz. This means that the tuning forks must be calibrated at a controlled temperature of 20°C, with a maximum deviation of ± 0.5 Hz. It may appear exaggerated to use such exact tuning forks, but experience has shown that tuning mistakes may otherwise easily occur. Great differences and changes in temperature may cause intonation fluctuation by as much as a minor second. Correct tuning is most easily achieved by proceeding according to Table 13, on p. 62.

1) The table shows the order in which the middle octave c^1–c^2 is to be tuned on a lyre, for example. When tuning be careful that all fourths and fifths sound pure. They will then have a light, transparent sound and their difference tones[13] will resonate 'silver' alongside.† If all fourths and fifths are perfect, the size of the formed fifths, true b:$gelis^1$ and belis:true f^1 will be correct and they will sound quiet and harmonic.
2) Fourths are easier to tune than fifths as the exactness of the tuning can be checked by means of the difference tone, which, when the fourth is tuned correctly, is one of the natural phenomena of the fourth. This difference tone sounds two octaves lower than the upper tone, for example:

Perfect fourths give difference tone resonating in the air



* See also chapters 22 to 24.

†The phenomenon of difference tones is dealt with in chapter 22.

The difference tone is so called because it sounds on the frequency which is the difference of the frequency of the two tones which make up the interval,[13] e.g.:

	f^1 = $341\frac{1}{3}$ Hz	g^1 = 384 Hz	a^1 = 432 Hz	e^1 = 324 Hz
	c^1 = 256 Hz	d^1 = 288 Hz	e^1 = 342 Hz	b^1 = 243 Hz
difference	F = $85\frac{1}{3}$ Hz	G = 96 Hz	A = 108 Hz	E = 81 Hz

With perfect fifths the difference tone sounds more softly, as it is the lower octave of the lower tone and they thus almost sound the same to the ear. Some examples are given below to show where these difference tones are to be heard:

	g^1 = 384 Hz	a^1 = 432 Hz	b^1 = 486 Hz	d^2 = 576 Hz
	c^1 = 256 Hz	d^1 = 288 Hz	e^1 = 324 Hz	g^1 = 384 Hz
difference	c = 128 Hz	d = 144 Hz	e = 162 Hz	g = 192 Hz

Some people initially find it difficult to hear the difference tone. With a bit of patience and practice this soon gets easy and one can then be certain that the fifths and fourths are correctly tuned.

3) Once the first nine steps have been completed, once again check that all fourths and fifths are perfect. If this is the case then the diatonic tones have been correctly tuned.
4) Now exactly tune $gelis^1$ = 362.04 Hz to the tuning fork. As $gelis^1$ is the exact middle, i.e., the geometric median between f^1 and g^1, and, if tuned correctly, semitones f^1–$gelis^1$ and g^1–$gelis^1$ will be exactly the same size, the major third d^1–$gelis^1$ will sound light, clear and pleasant, and the formed fifth $gelis^1$–b^1 quiet and peaceful. That an aurally genuine fifth exists which is smaller than the tempered fifth is a phenomenon that can only be discovered by ear. The still smaller 'grave' fifth 27:40[13] beats very strongly, and is therefore known to be unacceptable to the ear. The scale of twelve fifths only became possible with the discovery of the formed fifth. The formed fifth is recognizable by its somewhat raw or dry character. It differs from all other fifths that have been acoustically classified until now.
5) Once $gelis^1$ has been correctly tuned, the four other chromatic tones $delis^1$, $alis^1$, $elis^1$ and $belis^1$ are tuned as perfect fifths.
6) Finally the lower octave of $belis^1$, belis is tuned and the fifth f^1–belis checked. If the twelve diatonic and chromatic tones have been correctly tuned, this formed fifth sounds correct and is of the same size as the formed fifth $gelis^1$–b. This completes the tuning of the octave c–c^1.
7) The remaining octaves of the instrument are tuned next, based on the already tuned octave and proceeding in perfect fourths and fifths. Each newly tuned tone needs to be tested that it makes perfect fourths and fifths with the already existing tones. For example tune:

a descending perfect fourth e^1:a. Check that the fifth d^1:a is perfect;
a descending perfect fifth d^1:g. Check that the fourth c^1:g is perfect;
descending perfect fifths $delis^1$:gelis and c^1:formed. No need to check, as fourths b:gelis and belis:f are formed fifths;
descending fourths e:B and elis:Belis. No need to check, as fifths gelis:B and f:Belis are formed fifths;
ascending perfect fifth f^1:c^2 and check with fourth g^1:c^2, etc. up to the ascending perfect fifth a^1:e^2 and check the fourth e^2:b^1;
with ascending perfect fourths c^2:f^2 and $delis^2$:$gelis^2$. No need to check, as the fifths $belis^1$:f^2 and b^1:$gelis^2$ are formed;
the ascending perfect fifth c^2:g^2 and check with the fourth d^2:g^2 . . . up to the ascending perfect fifth d^2:a^2 and check e^2:a^2;
with the ascending perfect fifths $elis^2$:$belis^2$ and e^2:b^2 once again no need to check as the fourths f^2:$belis^2$ and $gelis^2$:b^2 are formed.

Experience has shown that it is easier to tune the descending before the ascending octaves. With lyres, tuning is easiest when the instrument is placed on a table and not on the lap. In this way one can 'set' the pin exactly without there being interference from involuntary movements of the body or instrument.

Table 13
Tone sequence for tuning the scale of twelve fifths

	Tone:	b^1	belis^1	a^1	alis^1	g^1	gelis^1	f^1	e^1	elis^1	d^1	delis^1	c^1	b	belis
	Hz:	486	458.206	432	407.290	384	362.04	341.333	324	305.470	288	271.530	256	243	229.100
1)	Tune:			a^1 = 432				and					c^1 = 256		exactly to the tuning forks
2)	Tune:							f^1			ASCENDING fourth		c^1		
3)	Tune:					g^1			ASCENDING fifth				c^1		
4)	Tune:			a^1				DESCENDING fifth			d^1				
5)	Check:					g^1			DESCENDING fourth		d^1				
6)	Tune:			a^1		DESCENDING fourth			e^1						
7)	Tune:	b^1			ASCENDING fifth				e^1						
8)	Tune:								e^1			DESCENDING fourth		b	
9)	Check:	b^1					octave							b	
10)	Tune:						gelis^1 = 362.04						exactly to the tuning fork		
11)	Check:						gelis^1			DESCENDING formed fifth				b	
12)	Tune:						gelis^1		DESCENDING fourth			delis^1			
13)	Tune:				alis^1				ASCENDING fifth			delis^1			
14)	Tune:				alis^1			DESCENDING fourth		elis^1					
15)	Tune:		belis^1				ASCENDING fifth			elis^1					
16)	Tune:									elis^1		DESCENDING fourth			belis
17)	Check:		belis^1					octave							belis
18)	Check:							f^1			DESCENDING formed fifth				belis

* The underlined tones indicate those tones from which the interval to be tuned proceeds.

Required material: Tuning forks tuned to c^1 = 256 Hz, a^1 = 432 Hz and gelis^1 = 362.04 Hz. The tuning forks must be gauged at a controlled temperature of 20°C with a maximum margin of error of ± 0.5 Hz.

8) When all octaves have been tuned, check by playing through all 24 major and minor arpeggios: C, a, F, d, B♭, g, E♭, c, A♭, f etc. If correctly tuned, all arpeggios without exception will sound, clear, beautiful and harmonic.

Pianos can also be tuned to the scale of twelve fifths. Our newly renovated (1966) and restrung Steinway grand piano built in 1866 sounds free and clear in every register. The richness in tone colour and timbre brought about by this tuning is unique and must be heard to be believed.

Listeners commented that the sound did not come directly from the instrument itself, but sounded freely from the centre of the room. A similarly free-sounding tone was observed with a choir of about 20 lyres tuned in this way; players and listeners experienced an inspiring fullness of sound that came from the middle of the room.

This tone coming from the middle of the room, which is felt to fill the whole space at once, may be called a 'free' tone. This characteristic brings to mind how Rudolf Steiner described a tone as 'etheric' in a talk with the American singer Gracia Ricardo (who died in 1955). He also said that she created this tone when singing. Regarding the singing method of this great singer, the long awaited book by her students Hilde Deighton, Gina Palermo and Dina Winter, *Singing and the Etheric Tone*, has at last been published.[81] Previously it appeared that only specially gifted artists, instrumentalists and conductors were able to create this free tone in active music making. The scale of twelve fifths appears to bring this free tone into existence simply by means of the interval proportions of the tuned tones. These interval proportions are due to the fact of the two rows of aurally genuine, harmonically usable fifths: the perfect and the formed. In a lecture given in March 1917,[41] Walter Blume showed that every interval degree of a scale is experienced in a distinct part of the astral body, the fifth in that part which relates to the spiritual soul. The printed edition of this lecture had a postscript added by Rudolf Steiner in which he wrote that Blume's application of spiritual scientific insights was absolutely right but could only be applied like this in music. Regarding the interval of the fifth, Rudolf Steiner himself said: 'The fifth is the real experience of vision in images. Whoever experiences the fifth correctly, knows what imagination is in subjective terms . . . the experience of the fifth is a real experience of vision in images.'[30] Regarding the entrance into the spiritual world through the fifth: 'The fifth will lead to more subjective experiences; it will have a stimulating and enriching influence on the inner life. It will work like a magic wand, conjuring up the secrets of yonder tone world from unfathomed depths.'[42]

Finally, reference shall be made to a key characteristic of the scale of twelve fifths. Just as the intervals and tones have a beautiful, pleasant and harmonic effect on the human being when tuned to $c^1 = 256$, $gelis^1 = 362.04$ and $a^1 = 432$ Hz, so they become equally antisocial, and indeed cause people to provoke one another, if the concert pitch $a^1 = 440$ Hz is used. This frequently made observation shows that aurally genuine intervals are not of sole importance in music, but that tones of certain frequencies have characteristic qualities that can have major effects on human beings. In the second part of this book, questions regarding the singular qualities of individual tones will be considered in more detail.

As a conclusion to this first part of our considerations, it may be of interest to the reader to gain an overview of all the intervals discussed so far. These are summarized in Table 14. The difference in cents between aurally genuine and false intervals clearly shows that the human ear can clearly distinguish between such intervals even if the difference in cents is very small. This is why a judgement as to whether or not an interval is genuine can never be based on calculation but purely on unbiased and exact human hearing. This may then be consolidated by the form principles of the scales. It was said in the introduction that electronic production of tones and intervals blurs the qualitative differences between tones (see chapter 15). Comparative experiments can only be carried out with appropriately tuned instruments such as tuning forks and exactly divided monochord boards built like the one for which instructions are given in Appendix 1.

Table 14
Summary of the intervals discussed so far

Interval	Just major and minor scales		Aulos modes		Formed		Equal-tempered twelve-tone scale		True scales	
	proportion	cents	proportion	cents	proportion	cents	proportion	cents	proportion	cents
semitone									256:343	90.225
							89:84	100+		
					67.88:64	101.995				
	16:15	111.731	16:15	111.731						
			14:13++	128.298						
			13:12++	138.573						
$^{3}/_{4}$ tone			12:11++	150.637						
			11:10++	165.004						
whole tone	10:9	182.404	10:9	182.404						
					8.94:8	192.180				
							449:400	200+		
	9:8	203.910	9:8	203.910					9:8	203.910
			8:7++	231.174						
minor third			15:13++	247.741						
			7:6++	266.871						
			13:11++	289.210						
									23:27	294.135
							44:37	300		
					76.27:64	305.865				
	6:5	315.641	6:5	315.641						
			11:9++	347.408						
major third			16:13++	359.472						
	5:4	386.314	5:4	386.314						
					80.45:64	396.090				
							63:50	400+		
									81:64	407.820
			14:11++	417.508						
	15:12	386.313								
			9:7++	435.084						

			9:7++	435.084						
fourth			13:10++	454.214						
	4:3	498.045	4:3	498.045					4:3	498.045
							303:227	500+		
	acute+++				4.07:3	509.775				
	27:20	519.551								
			15:11++	536.951						
tritone			11:8++	551.318						
			18:13++	563.382						
			7:5++	582.512					dim. fifth	
	aug. fourth								1024:729	588.269
	45:32	590.224			equal-tempered		equal-tempered			
	dim. fifth				1.4142:1	600	1.4142:1	600		
	64:45	609.777							aug. fourth	
									729:512	611.731
			10:7++	617.488						
			13:9++	636.618						
			16:11++	648.682						
fifth	grave+++		22.15++	663.049						
	40:27	680.449								
					2.9796:2	690.225				
							433:289	700+		
	3:2	701.955	3:2	701.955					3:2	701.955
			20:13++	745.786						
minor sixth			14.9++	764.916						
			11:7++	782.492						
									128:81	792.180
							100:63	800+		
					101.82:64	803.910				
	8:5	813.686	8:5	813.686						
			13:8++	840.528						

major sixth			18:11	852.592						
	5:3	884.359	5:3	884.359						
					45.26:27	894.135				
							37:22	900		
									27:16	905.865
			22:13++	910.790						
			12:7++	933.129						
			26:15++	952.259						
minor seventh			7:4++	968.826						
	16:9	996.090	16:9	996.090					16:9	996.090
							98:55	1000+		
					114.56:64	1007.820				
	9:5	1017.596	9:5	1017.596						
middle seventh			20:11++	1034.996						
			11:6++	1049.363						
major seventh			24.13++	1061.427						
			13:7++	1071.702						
	15:8	1088.269	15:8	1088.269						
					241.41:128	1098.045				
							168:89	1100+		
									243:128	1109.775
octave	2:1	1200+	2:1	1200+	2:1	1200+	2:1	1200+	2:1	1200+

\+ aurally false intervals*

++ intervals which are only melodically usable

+++ harmonically unbearable intervals

Abbreviations: dim. = diminished; aug. = augmented.

* See footnote† on page 8.

Part Two
TONES

* See footnote[7] on page 8.

14 The Individual Quality of the Single Tones and Rudolf Steiner's Concert Pitch Suggestion c = 128 Hz

So far we have been considering aurally genuine as opposed to false sounding intervals and their importance in music making. Now the pitch of individual tones, which is of even greater importance for human beings, and thus the height of our concert pitch, will be considered, with consideration above all given to a verbal suggestion Rudolf Steiner made to Kathleen Schlesinger and Elsie Hamilton regarding the concert pitch 'c = 128 Hz = Sun'.

The comment made at the end of chapter 13 that the scale of twelve fifths should not be tuned at the concert pitch $a^1 = 440$ Hz arose from the following experiences. The first time the author (concert violist and violinist) tuned the above-mentioned Steinway grand piano to the newly found tuning, the only tuning fork she had was $a^1 = 440$ Hz, and the piano was therefore tuned according to this pitch. Once this was done, it was an occasion to celebrate and music was immediately played on the instrument with its absolutely wonderful sound. Classical and modern works sounded in a beauty not heard before, but after a while an increasingly spiteful atmosphere developed amongst the people present.

It seemed totally improbable that the perfectly clear and harmonic sounding intervals of the new tuning method should arouse such an antisocial mood amongst the listeners. However, it did arise. The solution was only found when the piano was retuned to Rudolf Steiner's suggested pitch of $c^1 = 256$ Hz, the 'philosopher's C'. When more music was played on the retuned piano, it truly was a celebration. The spitefulness experienced earlier had gone and both intervals and tones sounded pleasant and beautiful. Everyone present was delighted at the splendid sound and felt sustained by a harmonic mood that left people free.

To make sure that the first, for musicians' unusual observation, was not a deception, the experiment was repeated with lyres over many years and in many places. The same phenomena always took place. The observation can lead to only one conclusion: it can only be the tones based on $a^1 = 440$ Hz that cause the antisocial mood.

Such a conclusion may be disconcerting and provocative, especially as it appears to be inescapable. Single tones and groups of tones that are less than a quarter of a tone different in pitch prove to have a very different effect on human beings, and the difference is such that the one causes a feeling of spite, the other good will. Truly an unnerving observation for an 'enlightened' modern musician to make, since it points to nothing less than the fact that good and bad principles are at work in the pitches of tones.[1,16]

It is interesting to note that Kathleen Schlesinger encountered a similar problem in her work with the aulos modes in the early twenties. She noticed that apart from its modal ethos, each mode would have a different effect on the listeners, depending on which tone was used to tune to. This was the reason why she asked Rudolf Steiner in 1921 on which tóne the 22/22 aulos Sun mode should be tuned, so that it would be right for the consciousness of the modern human being. His answer was: 'You need to use c = 128 Hz = Sun'.

Amongst the many indications which Rudolf Steiner gave for music, this one is singular in that it establishes the number of Hz, i.e., the absolute pitch for c. Schlesinger and Hamilton have used this indication and passed it on to their students and musician friends, and actually gave away tuning forks with c = 128 and $c^1 = 256$ Hz. Several people have since then continued to work with this C, among them the musician Else Goehrum in Stuttgart, as became apparent on a visit to her studio. Schlesinger's colleagues Wilhelmine Roelvink and Mary Wilbers in Holland passed this indication on to the author in 1948. As the indication had been mentioned by two independent sources it could be ascertained to be correct. That this pitch indication not only applies to the modes and just scales but also to the present scale of twelve fifths, and that it can be an undreamt-of blessing, will be shown in chapters 15 to 21.

The assumption that tones on certain frequencies exhibit very definite individual qualities that differ fundamentally from the qualities of other tones that lie very close by is nothing new. Music historian C. Forsyth wrote very positively about the moral value of Greek music.[16] The relevant passage is quoted below:

The 'moral value' of Greek music

We now come to a very difficult matter in connection with Greek music. Strange as it may seem, the Greek writers all agreed that music had a serious moral value. They did not say vaguely, as we do, that music was a beautiful thing and had an ennobling effect on the human mind. On the contrary, they said that according to the way in which it was written it was actually good or bad, that it had a definitely good or bad influence on the development of personal character, and that therefore the musical means employed was a matter of the most lively concern to educationists and statesmen.

This moral character, which they regarded as inherent in the art, was called the ethos of music; its value to society in general was known as its ethical value. Philosophers differed in explaining why there was an ethos in music, they differed also in discussing its practical application, but none of them ever dreamed of disputing its existence.

Now it is quite plain that all modern Europeans and Americans would agree that 'good music'—that is, music which is the honest expression of life viewed through the personality of a great composer—is better than 'bad music', which is just the opposite. But this was not the Greek attitude at all. Their constant subject of discussion was whether this mode or that mode was better suited for inculcating this or that form of moral excellence. In modern musical terms the ancient question may be put somewhat as follows: 'We wish to represent a brave man battling against adversity. Now, which is the better key for enforcing the idea of courage, E minor or C major?'

Let us examine this point. If we put our prepossessions on one side and admit that the question calls for a serious answer, we must agree that, whichever key we favour, our preference can only be supported on one of two grounds. The keys named differ only (1) in pitch, (2) in the arrangement of their tones and minor seconds. We must therefore justify our choice in one of two ways.

(1) We may say that a melody in the chosen key will, on the whole, lie at a higher (or lower) level of pitch than a melody in a rejected key; and that the difference of pitch makes the chosen key the better medium for the expression of manly courage.

(2) On the other hand we may say that pitch has nothing to do with courage or any other form of virtue; that the essential difference between the two keys is the difference between major and minor, that the varying moral values of scales are to be found solely in the varying arrangements of tones and minor seconds; and that therefore, whatever the pitch, we select major (or minor) as the proper medium for exhibiting the idea of personal bravery.

Nobody denies that the Greeks must have answered the question in one of these two ways. The only doubt has been 'in which?' Here our views were for long distorted by the fact that Greek musical practice was always examined through the lens of the Middle Ages. It was known that medieval priests and musicians consciously used their modes to express differences of feeling by means of differences in the order of the intervals. Scholars therefore jumped to the conclusion that the Greek mind worked in the same way. In fact as recently as 30 years ago, it was held that the Greeks would have answered the question in the second (or medieval) way. This was the orthodox German view. However, since then, an English scholar has clearly shown that—whatever the medieval prejudices in the matter may have been—every known reference to the question in Greek literature proves that the Greek himself would have made his answer depend solely on the pitch of the modes at his disposal. This is not the place to present the evidence or to go behind the whole difficult problem

> and ask why the Greeks associated a different ethos with a difference of pitch. It is enough to say that the evidence for this association is historically cumulative and overwhelming.
> But we must repeat here that the 'pitch' was a 'tonic pitch' and not a 'scale pitch' in our sense of the words.[16] [pages 63–5]

A further indication that in antiquity an exact feeling for absolute tones and the evaluation of a specific pitch must have been widespread is to be found with K. Schlesinger. She wrote:

> It may be recalled in this connection that the standard pipe known as Lu of Hoang Chung, on which the canons of ancient Chinese music were based, had as fundamental note an f^1 [fau^1] of 352 Hz, if we may judge from the dimensions of the pipe transmitted by Ancient Chinese authors. An F [Fau] of that vibration frequency occurs as 11th harmonic on a fundamental $_1C$ of 32 Hz. It may be added that it has been my frequent experience during research in the music of primitive folk to find their musical instruments tuned to an f^1 [fau^1] of 352 Hz and to c^1 of 256 Hz, also to find notes of those frequencies prominent in their songs.[8] [page 149]

Even today specific pitches can be seen to be important in folk music. A professor of classical languages at an American University who was keenly interested in old music and old instruments had built a lyre and a kithara according to Greek measurements and collected old and modern instruments from European and non-European countries. His collection included flutes built similarly to the recorder, which he could play, though he had no real knowledge of their tones and scales. He knew of Schlesinger's book, but did not agree with her statements regarding absolute pitches and even wished to make a written refutation. He had very good relative pitch but not absolute pitch. Using a monochord with the string fundamental tuned to c = 128 Hz it proved possible to establish the fundamentals and scales of several of his flutes exactly. This surprised him greatly and helped him overcome his scepticism towards the quality of different pitches. It may be of interest to the reader to know the scales of these flutes. They were all modal and are therefore given here in Schlesinger's terminology, together with the Hz of the tones:

1) Modern Korean flute
Scale of the flute:

Degree:
1 24/28 = 597.2 Hz = fundamental of the flute
2 22/28 = 651.6 Hz
3 20/28 = 716.8 Hz
4 18/28 = 796.4 Hz
5 16/28 = 896.0 Hz = mese = low $b\flat^3$ ('very small' seventh)
6 14/28 = 1024.0 Hz = c^3 = higher octave of the mode's fundamental
7 13/28 = 1102.4 Hz
8 22/28 = 1194.4 Hz = octave of the fundamental of the flute

This scale is identical to the Phrygian species of the 28/28 Mixolydian aulos mode and has the same fundamental and mese.

2) Ethiopian contemporary shepherd flute
(bought in Ethiopia from a shepherd boy who was playing it)

Degree:
1 c^2 = 512 Hz = fundamental of the flute

The other tones of the flute could not be ascertained as the mouthpiece was missing and it was therefore very difficult to play.

3) Modern Greek church flute
Scale of the flute:

Degree:

1 56/80 = 502.9 Hz = fundamental of the flute
2 52/80 = 541.5 Hz
3 49/80 = 547.7 Hz
4 44/80 = 640.0 Hz = just e^2 on c^2 = 512 Hz
5 40/80 = 704.0 Hz = fau^2, upper octave of the fundamental of the mode
6 36/80 = 782.2 Hz
7 32/80 = 880.0 Hz = mese = present-day concert pitch a^3
8 28/80 = 1005.7 Hz = octave of the flute's fundamental

This scale has the fundamental fau^1 = 352 Hz and the mese on a^1 = 440 Hz, one of the currently accepted concert pitches. Because it has the degree 49 instead of 48 this means it is an altered Mixolydian species of the 80/80 Hypolydian Mars aulos mode of the generic tone a^7 = 28,160 Hz which Schlesinger calls 'bastard Mixolydian species'.[8]

Without exception all tones of the flutes were in exact agreement with the scale pitches of the monochord and therefore had exactly the same frequencies. This exactness of intonation was especially noteworthy, as all three flutes came from rural areas and it was obvious from their appearance that they had been made by lay people. The makers of these flutes may therefore be said still to have had the gift of an exact sense for tones on specific frequencies, as mentioned above.

There must therefore still be an aural ability in the East which recognizes the inner quality of a tone. The observations concerning the scale of twelve fifths mentioned at the beginning of the chapter show that this ability is also to be found among Western people. This aural ability, which makes it possible to make a qualitative judgement of pitch, has proved to be very widespread. However, it has nothing to do with the peculiarity of perfect pitch, which is only to be found amongst relatively few Westerners. The important and luckily common ability to recognize the qualities of pitches should be taken note of and given due recognition again. The following two chapters include individual examples to show how the present-day existence of this ability can be established.

15 Perceiving the Ethos of a Tone

To consider this important phenomenon we must first establish the degree to which the human ear can discern qualitative differences in pitch. The quality and pitch between two tones as small as the interval of a Pythagorean comma can be easily distinguished even by lay people. The Pythagorean comma between just b (240 Hz) and true b (243 Hz) marks a difference of only 3 Hz. The difference between true b (243 Hz) and modal b (242 Hz) of only 1 Hz can be heard and experienced by trained musicians. Within the octave between b and $c\sharp^1$ an uncertainty in hearing first becomes apparent with intervals less than 1 Hz. Therefore the margin of error in the area of middle c^1 can be set at

± 1.0 Hz per second.*

An exact and simple order and method of experimentation is imperative if one wishes to make aural tests with people in many places and for many years, such as were mentioned in the previous chapter. The tones tested today need to be, as far as is humanly possible, the same as those of the week, month or year before.

Only with tones produced on musical instruments (tuning forks, monochord, chimes, wind instruments, piano, lyre etc.) is the individual quality of single tones observable. It might have been the obvious choice to use electronically produced tones. However, all experiments done in that way failed. The machines were of the highest quality but tones produced electrically, e.g. on a synthesizer or tape recorder etc., no matter at which frequency, all had the same levelling quality, and indeed a quality of yawning emptiness. It would be wrong, however, to draw the conclusion from this that electronic tones are easier for human beings because they eliminate the individual quality of tones. On the contrary, electricity puts its own quality on tones, and this is always there whether the sound is reproduced electronically or even merely amplified. It can be metaphorically described as a grinning vacuum.† In addition, the effect of yawning emptiness is greater the more perfect and static-free the reproduction is. This can be most clearly observed with 'sine tones'.[7]

Unfortunately the hollowed-out feeling which arises when listening to reproduced music is hardly ever noticed, as the attention of the listener is directed towards the composition and the interpretation and not the tonal quality of the reproduction. It is therefore all the more important that the characteristics discovered during the repeated aural experiments are brought to conscious awareness. The influence mostly remains unconscious and therefore goes unnoticed, so that its effects are all the more damaging. It is something that needs to be strictly taken into account with children, so that they do not suffer permanent damage in their bodies and souls at the tender age of childhood. Not for nothing did Rudolf Steiner say:

> . . . with the gramophone, people are trying to bring the mechanical into the realm of art. If humanity were to develop a passionate preference for such things, so that something which descends into the world as a shadow of the spiritual is made mechanical, if humanity were to develop an enthusiasm for such things, of which the gramophone is an expression, they would no longer be able to save themselves from it. Only the gods could then be of help.

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* In higher and lower octaves the margin of error is correspondingly similar.

† Eurythmist Dorothea Mier told the translator how she once performed in a modern theatre where, as is customary nowadays, the music was conveyed from the musicians to the stage via an amplification system. She asked for the system to be turned off. However, during the performance she felt as if the eurythmy was being torn away from her arms. It turned out that the amplification system had been turned on again. (Translator's note.)

> But the gods are merciful, and today it may still be hoped that with regard to the advancement of human civilization, the gods will be merciful and help humanity to overcome such aberrations of taste as come to expression in the gramophone.[43]

The monochord has proved to be the most reliable instrument for aural experiments as it has the great advantage of being exact. To begin with, the strings were tuned, as exactly as is possible for a practised human ear, with the help of a tuning fork. The tones were then played with a good cello bow. Special attention was paid to playing all tones with the same type of bow stroke and tonal quality.

As the experiments were a first attempt, they were initially limited to two pairs of tones based on the two pitches c = 128 Hz and a^1 = 440 Hz and their octaves. The two pairs of tones were therefore as follows:

1) c = 128.000 Hz and c = 130.813 Hz
(c^1 = 256.000 Hz and c^1 = 261.626 Hz)

2) A = 108.000 Hz and A = 110.000 Hz
(a = 216.000 Hz and a = 220.000 Hz)

c^1 = 261.626 Hz is the equal-tempered major sixth below a^1 = 440; a = 216 Hz is the true major sixth above c = 128 Hz.

Aural experiments were carried out with these two pairs of tones on more than 2000 people of all ages and of different occupations in the USA, Italy, Germany and Switzerland, and the results were recorded. Efforts were made to create an atmosphere that was as natural and unconstrained as possible both before and during the experiments. First the two outer strings of the monochord were tuned to the pair of tones to be observed and the listeners were told that one tone was tuned higher, the other lower. They were then told that the pitch of the tones was not important, but that we were concerned with the character of each, if they had the same or different qualities. The tones were then played one after the other as often as requested. The sequence was varied in order to avoid any influence due to pitch.

The cardinal question was: Did the two tones have a different effect, even though the difference in pitch was minimal? Were they experienced to have different and inherently individual qualities?

The results were extraordinarily interesting and unequivocal. Almost all of the people questioned said that the two pairs of tones and their octaves had unmistakably and individually different qualities for them as listeners. It was not easy for them to put these qualities into words as the experience was new and unexpected. But most of them had a more or less definite feeling for what those qualities were.

After one such aural experiment a mathematics professor asked whether the difference in quality between the two c-tones could lie in the structure of the monochord's resonance box and whether this favoured c = 128 rather than c = 130.813 Hz. Fortunately it was possible to answer this wholly justified question, as it was possible to use a small but very noble-sounding grand piano, tuned in equal temperament to a^1 = 440 (the c therefore being 130.813 Hz), that was in the next room. Our c^1 = 256 Hz tuning fork was used as a second instrument for the experiment. Both tones were played one after the other, as before. In spite of the difference in sound between the two instruments, the result was the same: each of the two c^1-tones retained its clearly distinguishable inherent quality, the same as experienced before on the monochord.

Over the course of the years many comparisons of this type were made, using, for example, home-made bamboo flutes, student violins and concert instruments, with the tones played on the better instrument and then on the less good instrument. The result was always the same, with each tone proving to have an inherent quality and that this quality remained constant no matter in which register or on what instrument it was played. We may therefore stipulate that the individual quality of the tones originates in the tones themselves and not in the instruments on which they are played.

The first result is that our aural experiments clearly showed that what Forsyth says about the tone-perception of the Greeks is still valid for present-day human beings.[16]

The intrinsic qualities of the tones that were experienced can also be brought in connection with Rudolf Steiner's fundamental scientific concepts. Having characterized the nature of such fundamental concepts as atoms, elements, force etc., he said the following about tone:[45]

> Neither tone nor warmth, nor light, nor electricity are waves, as little as a horse is the sum of its gallops. Tone, for example, is a qualitative element and the effect obtained when this element moves through the air is a wave. The wave induces sensitive individuals to imitate the quality within themselves; this means perception of the tone. It is similar with other things such as light, etc.

'Qualitative element' may be understood to be something similar to the experiences made in our experiments.

Having established that the chosen tones have definite inherent qualities, it was attempted, in spite of the expected difficulties, to find out from the listeners more exactly what the aural impressions of the pairs of tones were. This information is given in the next chapter.

16 Details of the Inherent Qualities of the Tones C and A (c = 128 and 130.828 Hz, A = 108 and 110 Hz) and Their Octaves

In the last chapter it was shown that the inherent quality of the individual tones of the named pairs was easily perceived by the greater majority of participants. Having distinguished the difference in pitch between the two tones, they were asked which of the two a- or c-tones they preferred. The answers were surprising. Although A = 110 Hz (220) was the most familiar tone, only 3–8 per cent of participants preferred it; c = 130.828 Hz (261.656) was preferred by even fewer. In other words, over 90 per cent of listeners preferred c = 128 Hz and a = 216 (the lower octave of a^1 = 432). The listeners were then asked to give the reason for their preference. The tone was played again as often as required. As the experiments were made over the course of 20 years and with very many people, it would be too much to report every remark again here. A summary of the many collected answers is therefore given.

Playing the single and double lower octaves of the present-day concert pitch a = 220 Hz and A = 110 Hz:

3–8% of listeners found that it
sounded beautiful,
had a stimulating effect;

over 90% found that it
sounded uncomfortable,
oppressive,
was irritating and very aggressive,
narrow-minded,
caused pain in the inner ear,
had an outward beauty, but that underneath it stabbed one with a dagger, appeared to want to have something to do with human beings but irritated, whipped and goaded them so that on hearing this tone they would have liked to have jumped up and laid to with their fists.

Hearing the tones A = 108 and 216 Hz (true major six above c = 128 Hz):

3–8% said that it sounded impersonal;

over 90% said that it sounded
correct,
complete,
peaceful and clear,
pleasing,
that it appeared to be outside the human being, but had a wonderful breadth which gave cause to a mood of devotion,
had light,
sounded Sunlike.

Hearing the present-day piano-C c = 130.828 Hz and c^1 = 261.565 Hz:

3–8% said that it
sounded happy,
light,

melodic,
soothing;

over 90% said that it
sounded biting,
narrowing,
cramping, slightly nasal,
irritating and unpleasant
like scratching the blackboard with one's fingernails,
cerebral, intellectual,
that it put a layer on one's chest which one first had to break through in order to be able to breathe freely,
caused disagreeable tensions,
drilled into the human being and went against his own rhythm,
made one nervous, assaulting the human being,
that the sound made one forebode evil, that it droned behind the ear and under the roof of the cranium, as if it wanted to force one out of one's head.

Hearing c = 128 Hz and c^1 = 256 Hz:

3–8% said that the tone
stimulated questions,
had an unsettling effect;

over 90% were of the opinion that it
belonged to the human being,
gave lots of space,
sounded peaceful, pleasant and full,
gave a sense of well-being,
harmonized with the human being,
took hold of a rhythm that was also in the human being and that one could go along with,
that on hearing this tone the human being could stand upright and breathe freely and deeply,
that it awakened trust,
was unassuming,
was experienced to be resounding in the heart,
was a primal tonal phenomenon to which the soul could relate other experiences,
sounded like a prime throughout the whole human being, yet left one totally free.

Considering that most participants found it difficult to describe the impressions gained on hearing the tones, the majority of 90 per cent is noteworthy. Below are some more detailed accounts given by teachers and musicians who had developed a more conscious attitude to music.

A violinist, who was over 80 years old, had absolute pitch and had played as a soloist in her youth and also founded her own string quartet, had been dissatisfied with a^1 = 440 Hz for a long time and therefore taken to tuning her instrument and quartet to a higher A. Compared to this A, a^1 = 432 Hz was very low. Once she had got used to it her response was: 'I could get to like this a = 216 or a^1 = 432 Hz as concert pitch for practice and also in performances.'

Independently of each other, an elderly violinist and teacher, a young student and a therapist immediately realized that a^1 = 432 Hz was the tone to which they habitually tuned their instruments when they played on their own.

A concert violist who until then had used the customary concert pitch had often noticed when teaching adults and children that many tended to suffer from nervousness and lack of concentration. He therefore tuned all instruments, including his own, to c = 128 Hz and a^1 = 432 Hz. The result was that a lot of the nervousness disappeared and that the students were able to make music in greater harmony.

A two-year-old boy (not yet a musician or teacher) who was able to walk but not yet talk and was present at one such aural experimentation, sat peacefully and happily with a clear gaze beside his parents while the tones c = 128 Hz and c^1 = 256 Hz, A = 108 Hz and a = 216 Hz were played. When A = 110 and a = 220 Hz, and c = 130.813 and c^1 = 261.626 Hz were played, he displayed definite discomfort, getting more and more restless and finally protesting against these tones by stomping loudly around the room.

A eurythmist said that the tones c = 256 and a^1 = 432 Hz harmonized with the angular movements for these tones, whereas c^1 = 261.626 and a^1 = 440 Hz were significantly less correct.

Two curative teachers who were anthroposophists were familiar with the concert pitch a^1 = 440 Hz. Having taken part in the experiments and heard the tones A = 108 and a = 216 and c = 128 and c^1 = 256 Hz they were so shocked by the goading quality of A = 110 and a = 220 Hz that they resolved from then on to tune their instruments to c = 128 Hz.

A Slavonic musician who was used to a c^1 that was the lower sixth of a high concert pitch between a^1 = 444 and 446 said, on hearing c = 128 and c^1 = 256 Hz for the first time, that the higher C might be more pleasing but that the lower C was good, meaning good as opposed to evil.

A further series of important experiments was the following. About a dozen people who tested c = 130.813 Hz and c = 128 Hz, and then decided to tune to the latter, were wondering if a C, which was even lower than c = 128 Hz, would sound even better. Participants were requested to tune the third string of the monochord, which had not been made use of in the experiments until then, to a C lower than 128 Hz that they felt to be comfortable. Each had to find this tone purely by ear. It was interesting that in spite of the differences among them, different localities and dates, they all chose a tone of the same pitch and quality. It was agreed amongst the tuners and other listeners present that this C, which was later calculated with the help of division of the monochord, was of a different quality to the other two C-tones. This new, lower C encouraged a pleasant bodily ease, and had a somewhat dreamy effect on the mind, though at the same time it made one calculating and merciless. Most of the tuners were quite shocked at the unexpectedly nasty effect of this tone.

Having marked the position of the C on the monochord board, the attempt was made to work out its frequency. After much searching it was found that its fourth higher octave was the ninth overtone of the mese of the 56/56 Mixolydian aulos Moon mode on the fundamental c = 128 Hz, i.e., the second upper fifth of the low just $b\flat^1$ = 224 Hz. The frequency of this tone is 2016 Hz, which is the same as c = 126 Hz. This tone is therefore roughly as much below c = 128 Hz as c = 130.813 Hz is above it. If 224 Hz is used as concert pitch, c = 126 is part of the tonal material of this pitch.

The example of this C shows once again that the character of tones that are only a few Hz removed from each other is very different. The experiments also substantiate the exceptional position of c = 128 Hz and its octaves. If one can convince piano tuners to use c^1 = 256 Hz as tuning pitch, they are generally surprised to find that the piano sounds so much more beautiful, even with equal-tempered tuning.

Independent of the author, several musicians and teachers have made similar observations. Their findings agree with our own and show that they arise from the phenomena themselves and are not caused by conscious or unconscious influence from the author.

The lyres had just been tuned to the scale of twelve fifths for the first time and the music therapist was making music with a patient when the patient's physician entered the room. She remarked spontaneously that the instruments sounded very different and that she experienced a harmonizing effect on her body and her breathing.

An English curative teacher who did a lot of unaccompanied singing with about 400 children and adolescents noticed that all of his pupils managed the melodies without difficulty and returned correctly to the beginning tone when it was c^1 = 256 Hz, which was by no means the case when the tone was a^1 = 440 Hz.

A lady who had taught the piano for many years and had been an opera singer had tuned her piano to c^1 = 256 Hz for over 20 years. She had found that with this tuning pitch her vocal talent, which through age or incorrect singing had lost its natural elasticity, soon became pliant again and regained its youthful freshness. Tuning to c = 128 Hz makes it possible for the human organism to balance out different types of existing disharmonies better and more quickly. A voice schooled in this way can easily adapt to and master the problems connected with public singing and present-day tensed tuning.

The conductor of a lyre orchestra with 20 musicians noticed that the twelve fifth-tones tuning on c^1 = 256 Hz kept the players together unbelievably well throughout long rehearsal sessions. This tuning has less shine to it than that on a^1 = 440, but it sounds more true.

Our experiments show that tones have a moral value also for modern people. Summed up, the answers of the great majority of people questioned and comments made by musically trained people show that not only can we speak of the ethos of a tone but must do so.[16,45] Good as well as bad principles or influences assert themselves at the different pitches, as has also been shown for other areas of music by Carl von Balz.[1] A tone such as a^1 = 440, though beautiful, has a goading and antisocial effect. Such an effect can only be described as the antithesis of good. Other tones, such as c^1 = 256 Hz, have a harmonizing and beneficial effect and may therefore be called 'good'.

The frequency of the concert pitch in all aulos modes is therefore of great importance and must be chosen with care. Every concert pitch takes the human being to a multiplicity of tones, with the quality of the concert pitch determining that of all the other tones. If one enters the tonal world through a^1 = 440, all tones mirror its beautiful but goading quality, so that one unheedingly passes by tone-worlds of very similar frequency that are close by and beneficent.

The tendency to raise the concert pitch to 448–60 Hz, which has developed in most Central European orchestras since the last war, can have grave consequences. The tone 448 Hz is the 'very small' just minor seventh of the fundamental c^1 = 246 Hz and therefore the mese of the Mixolydian aulos mode, while 460 Hz is a 'normal' minor seventh of the fundamental c^1 = 256 Hz that is somewhat too low. Both tones should be therefore called 'B♭' and not 'A'. If one uses these tones as concert pitch, one leaves the region of the sixth of the fundamental C and enters that of the minor seventh.

A valid judgement of the inherent qualities of the tones that have been described can only be gained by hearing them and letting them work on one. They must be played on a non-electrical instrument. Reading about them or just doing pure calculation is not sufficient. A monochord can be used to test what has been presented here and also for further experimentation. To help with this, a tuning fork with the frequency c^1 = 256 Hz is supplied with this book. See also Appendix 1, p. 172, for instructions on how to use a tuning fork. Comparison may be made in the following way.

Listen to the c^1 = 256 Hz tone of the tuning fork, then play middle c^1 = 261.626 on the piano. Repeat this several times, changing the sequence of the tones. Having got used to the tones, try to listen for whether both tones only differ in pitch or if there are different qualities. Finally, try to bring the different qualities clearly to mind and when possible describe them in words.

17 The Human Being and the Tone c = 128 Hz

Now that the aural experiments have shown how c = 128 Hz is experienced by human beings, the author sees it as one of her most important tasks to establish the uniqueness of this tone. To assist in this, several aural indications given by Rudolf Steiner will be considered. As far as we know these have not appeared in print before.

To begin with, some examples from the literature of the twentieth century will be given. The great composer Paul Hindemith, who had an exceptionally accurate sense of absolute pitch,[46] based the chromatic scale recommended by him, which consists mainly of just intervals, on the tone C = 64 Hz. His only reason for doing this was that this tone is the 'standard measure for physical investigations'.[47] Roughly a decade after the publication of the second edition of Hindemith's *Unterweisung im Tonsatz*[47] (textbook of composition), Ernst Bindel wrote about absolute pitches.[11c] Based on Hindemith's theories he derived normal tuning for the concert pitch a^1 from the tone c^1 = 256 Hz. He calculated this to be the just major sixth 5:3 from c^1 and identified the pitch as a^1 = 426.667 Hz. If one stays with the just intervals, as Hindemith did in his chromatic scale, the a^1 = 426.667 Hz on the fundamental c^1 = 256 Hz is the correct concert pitch. It must however be stressed that with equal-tempered tuning on c^1 = 256 Hz the concert pitch would be a^1 = 430.541 Hz (a proportion of 1:1.6818 to c^1). If one prefers true-tone tuning, the concert pitch is a^1 = 432 Hz (proportion 27:16). Starting from c^1, three different concert pitches thus arise, depending on which kind of tuning one chooses. The different qualities of these concert pitches can be heard if they are played on a monochord (see also chapter 21, pp. 107–22).

In the same work Bindel abandoned c = 128 as the fundamental and tried to show that, based on the human rhythmic system (breath and heartbeat), the actual musical fundamental should be an 'E♭' and not a 'C'. Taking the average rate to be 18 breaths and 72 heartbeats per minute (60 seconds), one gets a fundamental for the rhythmic system of $_6$E♭ = 1.200 Hz (60:72 = 1:1.2). In the octave from c = 128 to c^1 = 256 this corresponds to the tone e♭ = 153,600 Hz—the ascending just minor third 6:5 to c = 128 Hz. Basing himself on the tone eurythmy course where Rudolf Steiner described the connection between the skeletal structures in the human limbs and the intervals,[33, 50] Bindel then attempted to bring the other tones into a natural relationship to the human being. This attempt must be given due merit.

Bindel was not the only one to search for a new fundamental for our whole system of music. The singer Werbeck-Svaerdstroem nursed similar ideas and asked Rudolf Steiner if a tone other than C could be the basis for our system of music. His answer, passed down by Juergen Schriefer, was: 'No, the tone C must remain the fundamental of our system of music, otherwise people would stutter.'

This statement, curious at first glance, must be connected with Rudolf Steiner's spiritual-scientific views of the human organism. He described how the spiritual seed of the human larynx was created between death and a new birth in the sphere of Mars.[48] The human organ of speech is thus directly connected with the planet Mars. A year earlier Rudolf Steiner had described how the ear and the larynx related to one another.[49]

Two further indications were given by Rudolf Steiner in connection with the human ear. Regretfully both have only been passed on by word of mouth. The first was to the singer Werbeck-Svaerdstroem (passed on by Nanda Knauer): 'The inner ear of the human being is built on c = 128 Hz; the human middle ear beats at c = 128 Hz.' The second indication, passed down by Mary Wilbers, is similar: 'The human cortical organ [of the ear] is built on c = 128 Hz.'

In connection with this it may be remembered that ear specialists still use the tone c = 128 Hz today. Large medical supply shops sell tuning forks of c = 128, 256, etc. Hz, which are used for medical examinations. None of the many doctors questioned by the author could give a reason for the use of this particular frequency.

In 'answers to questions' after the lecture on 'Earthly and Human Development' of 17 March 1908 in Munich, Rudolf Steiner gave the following relationships of the seven non-altered tones to the planets and metals:[51]

C	iron	Mars
D	mercury	Mercury
E	tin	Jupiter
F	copper	Venus
G	lead	Saturn
A	gold	Sun
B	silver	Moon

The relationship of the planet Mars to the tone C helps us to understand the connection between the tone C, the larynx and the organs of hearing (Mars being connected to the organ of speech). Only the C and the A of the seven named tones could be investigated in the experiments presented in chapter 16. From the results given there it follows that a^1 = 432 Hz may be considered to be the Sun-gold-tone.

The relationship between the tone C, Mars and iron is especially important as we continue with our considerations, for in about 1921/22 Rudolf Steiner commented to Schlesinger that she should place 'c = 128 Hz equal with the Sun' (see chapter 14). Why? How do the tone c = 128 Hz, the Sun, Mars and iron relate to each other? On 17 November 1923 Rudolf Steiner threw light on this problem in a most impressive manner. The relevant part of the lecture is therefore quoted in full:[48]

> You are familiar with the phenomenon of sunspots appearing on the Sun with a certain regularity. There is much dispute among modern scientists as to the cause and significance of these sunspots. Yet if we were to consider the significance of the sunspots in more detail, we would find that an impulse is continually given from within the Sun, with Sun substance cast out into the cosmos through these dark portals. This Sun substance appears in our solar system in the form of comets, meteors and shooting stars. The spirits governing the world within the Sun in our time in particular are casting these things into our age. They also have done this before, and these things have not just happened in our time, but they now have a different significance. Because of this I said that in earlier times the spiritual impulses that were at work existed primarily in the stellar system. Now these impulses, which lie in the iron that is cast out, begin to have special significance for human beings. They are the impulses which a particular spirit, a spirit who is also gaining special significance in our time and whom we call the Michael spirit, is using in the cosmos to serve the spiritual in the cosmos. For our age, therefore, something has happened in the cosmos that did not exist to the same degree before. It is that cosmic iron in its spiritual significance makes it possible for the Michael spirit to mediate between the supersensible and the sense-perceptible spheres on earth. We thus have on the one hand a warlike mood in the world into which we enter when we get behind sense-perceptible existence today. When a human being develops supersensible vision today, when he crosses the threshold and now turns his gaze not towards things of immediate personal concern but to things that are major world concerns and lie at the foundation of the whole of our civilization, his gaze will penetrate into this world and perceive warfare, battle, spiritual struggle. Battles and struggles go on behind the scenes of existence in the spiritual world. The iron cast into the cosmos by the Sun spirits, to the point of physical distinctness, will then be the cosmic armour, in the widest sense, of Michael whose task in this cosmic battle is to help humanity to go forward in the right way in the face of these powers of battle and strife that are behind the scenes of civilization. On the one hand, therefore, we have war and strife, and on the other the efforts made by Michael.
>
> But all this is in turn connected with the evolution of human freedom. For you see, we have iron in our blood. If we were beings without iron in our blood, the feeling of freedom, the freedom impulse

could arise just as easily, but we would never have a body that we could use to make the impulse for freedom reality. The fact that we are not only able to conceive the idea of freedom but can also feel the power in our bodies to make them the vehicles for the freedom impulse arises from the fact that we are able to learn in the present age how Michael is able to make the cosmic iron, which was also cast out in former times, serve him. If we come to understand the Michael impulse more and more we can also learn to make the inner iron in us serve the freedom impulse. The material principle outside always only gains meaning if we come to see it as a reflection of the spiritual principle in the world. And we must learn in our age to use the iron in our blood in the right way, for wherever iron appears the impulse is given, out of the cosmos and out of the human being, for freedom to develop. Out of a deep instinct, the initiates of old therefore related iron to Mars; with its importance in the blood iron was thus also given significance in the cosmos.

The above makes it possible to see how the tone c = 128 Hz not only relates to the planet Mars and its metal iron but also to meteoric iron as Sun substance in the cosmos, to Michael as the spirit of the age, to human blood and human freedom. Our modest aural experiments have also shown show that c = 128 Hz and its octaves are deeply rooted in the human being, are beneficial to him and leave him totally free.

The special position of c = 128 Hz can also be found in a completely different area. A second as time unit is not to be found as a natural phenomenon and is therefore seen as an arbitrary measurement in science. Now c = 128 Hz is the seventh octave of $_6C$ = 1 Hz, which takes one second to beat once. It is not actually possible for the human ear to experience this, as the lowest sound that can be experienced as a tone is at about 16 Hz. Physical inaudibility does not, however, preclude the possibility of it having an effect. Our second in time therefore is by no means an arbitrary unit; as the measure of time for a lower octave of the tone c = 128 Hz it is rather—like the tone itself—deeply rooted in the human being and therefore creates a real connection between the human being and music, in so far as both are part of the stream of time.* A main characteristic of music is that it does not exist in space but only moves in time.

Concerning the future importance of the tone C, Rudolf Steiner said:[30] 'One can say that the "F" has already joined the five ancient tones D, E, F, G, B to a very high degree, but not yet the actual "C". This still has to first gain its full significance in human sentience.' His indication to Schlesinger, that 'c = 128 Hz is the Sun' can thus been seen as a gift by which he opened the way for Western humanity to this 'actual C'. This will be considered further in Part Three.

* A real connection between our measure of time and the human being is to be found in the 'Platonic Year'. A Platonic Year is the amount of time it takes for the point at which the Sun rises in the spring equinox to move through the whole zodiac. This takes 25,920 years. This figure is found again in the average number of breaths an adult person takes in 24 hours—18 breaths per minute × 60 minutes × 24 hours = 25,920 breaths. The old Babylonian year also had 360 days × 72 years (a human life span, from the esoteric point of view) = 25,920 days. In other words, we breathe 25,920 times a day and our life has 25,950 days. See Rudolf Steiner's lecture of 17 September 1924 and bibliographic reference no. 89. (Translator's note.)

Part Three

THE TONES IN THE SCALES

18 The Tones of the Aulos Sun Mode and the Tone c = 128 Hz*

In Part One it was shown that the just major and minor scales arise from the aulos modes, and how important the modes are for understanding these scales (chapter 9). The two statements by Rudolf Steiner given below will emphasize the importance of these aulos modes and the pitches of their scale degrees for contemporary musical life.

On 5 January 1922, during the Christmas course for teachers,[55] Rudolf Steiner was asked: 'Do you consider that a real progress for the future of music would be achieved by composing in the Greek scales discovered by Miss Schlesinger and tuning instruments, for example the piano, accordingly? Would it be helpful to accustom ourselves to these modes?' The answer was:

> I would like to say on this point that in view of the different conditions that pertain, my view must be that music will make a kind of advance in so far as the intensive melody, as I would call it, will play an ever-increasing role. Intensive melody would consist in accustoming oneself to experiencing as a kind of melody what today is seen as just one tone. People will get accustomed to a greater complication in the tones, in a single tone. This will happen. And when it happens, it will mean a modification, in a sense, of our scale, the simple reason being that intervals will be filled out in a different way than has been accepted until now. They will be filled out more concretely. And then people will actually in this way, I believe, make a connection with certain elements of what I would like to call original music, and I believe I see something of major importance relating to this in Miss Schlesinger's discovery of the modes. I do, however, think that a path is opening up here for the enrichment of musical perception in general and the discovery of certain things that will overcome what simply has entered into music because of our, I would say, more or less accidental scales. So yes, I do believe that it has a certain prospect if these specific discoveries are followed up further and if we get accustomed to these modes in our musical perception.

To allow modern human beings to work safely with the modes, it is necessary that, as well as taking care with the intonation of intervals, attention is also paid to the pitch of the single scale degrees.† This had been observed by Schlesinger who, as we have seen, asked Rudolf Steiner which was the correct pitch for today, his answer being that c = 128 Hz = Sun was the correct pitch for modern human minds and spirits. The generic tone of the Sun mode is the small modal tritone 8:11 which Schlesinger called the harmonic fourth.[8] Because the generic interval is made up of two tones Rudolf Steiner's pitch indication can be applied in two ways: c = 128 Hz as the fundamental and also the mese of the mode.

If c = 128 Hz is used as the fundamental, the generic tone is fau^4 = 2816 Hz (an upper octave of fau = 176 Hz). If on the other hand c = 128 Hz is mese (a lower octave of c^5 = 4096 Hz), then all tones of this mode lie on the undertone row of this C, and the 11th undertone geo = 186.182 Hz becomes the fundamental (see Table 5). As the degrees of a mode are made up of an octave section of the generic tone's undertone row, their character is determined by the inherent quality of the generic tone, whilst the inherent qualities of the other tones contained in the mode, including that of the fundamental, gain only minimal significance.

It is very informative to observe how the two methods of tuning the modes influence modern people. Years of observation have led to the following. If one tunes the modes to the fundamental c = 128 Hz

* A review of chapter 8 may be of help in understanding this chapter.

† The modes have a very powerful effect and work into the human physical and rhythmical organization. The author related to me how modes on certain pitches caused *excessive* menstrual bleeding in some women. (Translator's note.)

and mese fau = 176 Hz, as Schlesinger did, this creates a kind of ecstasy amongst listeners and a degree of somnambulance on extended listening. Ernst Bindel reported this earlier.[11a] It may be put down to the quality of a solemn rolling boil, strongly sweeping the human being along with it, in the tone fau = 176 Hz and its octaves. This method of tuning the Sun mode was very much suited to people of another age, when the old, natural clairvoyance had already faded away to a large extent. With the help of the degrees of an ecstasy-inducing Dionysian Sun mode it may still have been possible for them to dimly sense their connection to the spiritual world. For twentieth-century people such a dulling effect on the mind is definitely not desirable. Made free and independent through the Christ impulse, modern people need to find their way to the spiritual world with a clear, sober and matter-of-fact mind. This applies just as much to arranging one's life as to music making.

No effect on the mind of modern-day listeners has been observed with the Sun mode tuned to the mese c = 128 Hz or c^1 = 256 Hz and the fundamental to geo = 186.182 Hz, i.e., the generic tone on c^5 = 4096 Hz. This method of tuning actually leaves listeners totally free, letting them handle music and configure the mode in a completely free way. This second method of tuning the Sun mode seems more suitable for our time. It therefore also formed the basis for our summary of the modes in chapter 8 and Table 5.

In theory, this second method appears to be a species of the 32/32 Saturn mode on the fundamental and mese c = 128 Hz. On hearing it, one immediately realizes that this is not the case, however, for one experiences such a mode as a fully valid and independent scale. What is more, not only the Sun mode but all other six planetary modes sound like original, independent modes on the degrees of the generic tone c^5 = 4096 Hz. The relevant planetary degree tone becomes the fundamental of its own mode in the process (see lists on p. 33 and below). On the other hand, if one plays the modes on the degrees of another planetary generic tone of the common fundamental c = 128 Hz, for example on that of the Venus mode with g^4 = 3072 Hz, the Mercury mode with high $a\flat^4$ = 3328 Hz or even the Moon mode with low $b\flat^4$ = 3584 Hz (see tables on p. 29), one finds that only the mode belonging to the generic tone sounds authentic and that all others sound like species or transposed modes. From this it follows that the degrees of the generic tone c^5 = 4096 Hz, which belongs to the Sun mode, are also the rightful degrees for the six other planetary modes as independent scales. Once again readers are referred to the monochord; with this, one can easily convince oneself of the above.

The importance of the relationship specific tones have with the single degrees of the modes for today was also raised by Rudolf Steiner. Mary Wilbers, Wilhelmine Roelvink and Wilhelm Doerfler have told the author that for the row of planetary degrees between c^1 = mese = 256 Hz and c = mese = 128 Hz, in a conversation with Schlesinger Rudolf Steiner agreed with her on the connections made for the other degrees of the scale with the planets. This is given below. For the sake of completeness the author has added the descriptions just, true and modal and the Hz in parentheses. Wilbers and Roelvink were close friends of E. Hamilton, and Doerfler had studied in London with Schlesinger.

Second indication of Rudolf Steiner to Schlesinger

c^1 = mese	=	256.000 Hz	=	16	=	Saturn degree
[true] b♭	=	[227.555 Hz]	=	18	=	Jupiter degree
[just] a♭	=	[204.800 Hz]	=	20	=	Mars degree
[modal geo] g	=	[186.182 Hz]	=	22	=	Sun degree
[true] f	=	[170.667 Hz]	=	24	=	Venus degree
[low modal] e	=	[157.538 Hz]	=	26	=	Mercury degree
[high just] d	=	[146.286 Hz]	=	28	=	Moon degree
[just] d♭	=	[136.533 Hz]	=	30	=	Moon degree
c = mese	=	128.000 Hz	=	32	=	Saturn degree

The generic tone of these mese is c^5 = 4096 Hz. From this generic tone and its undertones we can derive the tones with which the modern human being can work safely using all seven aulos modes and so enrich the expressive potential of music (see Table 5 and Table 34, fourth line, 'blue', in Appendix 1). Schlesinger must have received the two indications mentioned in this chapter at different times, otherwise the fact that she continued with the questionable method of tuning to the generic tone fau^4 = 2816 Hz is not understandable.

According to Rudolf Steiner, c = 128 Hz and its octaves are the Saturn degrees appropriate for our age. In the previous chapter we were able to show how his indication that 'c = 128 Hz = Sun' gives us an access to the C tone, which he calls 'the actual C' which 'must still find its full significance in human sentience'.[30] This confirms the connection of the tone c = 128 Hz to Mars, meteoric iron, Sun substance and the degrees of the Saturn mode, and its great importance for modern music-making.

If one wishes to tune an instrument to the degrees of the seven aulos modes from the generic tone c^5 = 4096 Hz, this is possible with and without the use of a monochord. Using a monochord one tunes the string to c = 128 Hz and divides the string into 32 equal parts (see Appendix 1, Table 34, fourth line, 'blue', 32/32 Hypodorian). True middle c^1 (which is the degree 16/32) then sounds at 256 Hz; true b♭ (18/32) at 227.555 Hz; just a♭ (20/32) at 204.800 Hz; modal geo (22/32) at 186.182 Hz; true f (24/43) at 170.667 Hz; low modal e (26/32) at 157.538 Hz; high modal d (28/32) at 146.286 Hz; modal d♭ degrees (30/32) at 136.533 Hz; true c (32/32) at 128 Hz, the fundamental of the monochord string. The other octaves are gained by tuning to octaves. Without a monochord the above tones can be found with the help of a tuning fork at c = 128 or c^1 = 256 Hz and tuned by ear according to Table 15. This gives the nine modal tones between c and c^1. The tones of the next higher and lower octaves are found as above by the tuning in octaves. All seven aulos modes in Table 5 can be played on an instrument tuned in this way.

As examples of modal composition the 'Hymn to the Sun' composed by Schlesinger and referred to at the end of chapter 8, and the beginning of the Pindar's 'Pythian Ode' (despite doubts as to the authenticity of the latter) are given; both are based on the generic tone c^5 = 4096 Hz and can therefore be played on a lyre tuned in the above way.

Table 15 Tuning the aulos modes by ear

Proportion:	16/32	18/32	20/32	22/32	24/32	26/32	28/32	30/32	32/32
Tones:	true c^1	true b flat	just a flat	modal geo	true f	low modal e	high just d	just d flat	true c
Hz:	256	227.555	204.800	186.182	170.667	157.538	146.286	136.533	128
1) tune:	16/32 256	perfect octave with the tuning fork							32/32 128
2) tune by ear:	16/32 256	descending perfect fifth			24/32 170.667	ascending perfect fourth			32/32 128
3) tune:	16/32 256	just major third	20/32 204.800	just minor third	24/32 170.667				
4) tune:			20/32 204.800	descending perfect fifth				30/32 136.533	
5) check:					24/32 170.667	check: just major third		30/32 136.533	
6) tune:		18/32 227.555	ascending perfect fourth		24/32 170.667				
7) tune:	16/32 256	small tritone 8:11		22/32 186.182	extra large tritone 11:16				32/32 128

Tuning modal geo by ear is not difficult as it lies higher than true sharp and therefore also higher than true gelis.
Modal geo to the higher 'middle' c^1 forms the questioning sounding interval of a small tritone, called the harmonic fourth by Schlesinger.

	16/32	18/32	20/32	22/32	24/32	26/32	28/32	30/32	32/32
8)						26/32 157.538 small minor third 13:15 = 1¼ tone step		30/32 136.533	
9) check:					24/32 170.667 large semitone 12:13	26/32 157.538			
10) tune:							28/32 146.286	large whole tone 8:7	32/32 128

Each tone to be tuned is underlined. Left/right direction for the lyre

Hymn to the Sun*

Melody by Kathleen Schlesinger transposed by the author onto the degrees of the generic tone c^5 = 4096 Hz (mese c^1 = 256 Hz)

* The original with the words set on the mese fau is on p. 38.

Pindar (518–446 BC)
Melody to the first 'Pythian Ode'*

Golden phorminx, possession of Phoebus and black-haired Muses, precious jewel, its sound obeys the festive step which begins the merry dance, its beat is listened to by many a singer, when quaking at the strike of the first tone of your choir-leading strings, you build and destroy the wild spear of lightning, streaming in eternal fire.

* Transposed by the author onto the degrees of the generic tone c^5 = 4096 Hz.

19 The True C Major Scale* and Two Indications Rudolf Steiner Gave to Wilhelm Lewerenz

An attempt will now be made to show how the true C major scale† with the fundamental c = 128 Hz and two indications Rudolf Steiner gave to Wilhelm Lewerenz complement one another and can prove fruitful in the future development of Western music. Cellist and composer Wilhelm Lewerenz (1898–1956) came to Dornach in 1923, became a member of the Council of the General Anthroposophical Society and was leader of the Section for the Performing Arts from 1949 until his death.

Indication 1 came to light in the estate of Anny von Lange. Her publisher writes that Anny von Lange attempted to work out a centering of human beings in their thinking (Jupiter) and in the 'I' (Mars) based on this indication in connection with Rudolf Steiner's occult physiology.[17] As will become apparent in what follows, our approach is based on completely different premises.

Indication 2 only exists on a small scrap of paper on which Lewerenz noted it down. He often made use of this indication in lectures and courses and a number of anthroposophical musicians therefore have a photocopy of it. Unfortunately no record has been found until now amongst Lewerenz's notes that would tell us the context in which the two indications were made. An indication as to the exact Hz of the tones is also missing, perhaps because Lewerenz, in contradistinction to Schlesinger, had not noticed the varying inherent qualities of tones with the same names on different pitches and therefore did not ask about their frequencies.‡ The indications to Lewerenz thus have to be considered in the context of the rest of Rudolf Steiner's indications.

Table 16

Indication 1 has an upper and a lower part (here called 1a and 1b respectively), both with the same titles (second scale, difference, cosmic scale) and the comments 'etheric' and 'astral' to the left and right. The right-hand column, 'cosmic scale', is the same in both parts and gives the descending row of thirds G, E, C, A, F, D, B. The scales of seconds on the left of both parts are different, however, as the intervals of the middle difference column differ. Two tone-planet concordances are given in both parts—in 1a B = Moon and E = Jupiter, in 1b E = Jupiter and C = Mars. These agree with the concordances referred to in chapter 17 which Rudolf Steiner gave in answer to a question in 1908. The author took for granted that these concordances were also valid for the rest of the tones of Indication 1 and therefore added these into Tables 17, 18 and 19 accordingly.

Indication 2 has the seven octave scales, which go in descending direction, as shown by the arrow. Read vertically, the initial tones are identical with those of the second-scale in Indication 1a. Planets and human physical organs are given to the right of the initial tones of Indication 2. Read vertically, this planetary row corresponds with the tones of the cosmic scale in Indication 1, but not with the initial tones of the scales they are next to (see Table 17, chapter 8 and p. 81). Finally, a note added to Indication 2 says that in late Roman times the Lydian C was the normal proportion and that, as the lowest tone, it is to be related to the planet Saturn. Agreement between the tones of the second-scales of Indication 1 and the beginning tones of the octave scales of Indication 2 shows that the two Indications are connected, yet the two Lewerenz Indications are very contradictory. An interpretation will be attempted in the following paragraphs that may help towards understanding the development of music from antiquity until today.

* The Greek/Roman true scales were, in opposition to the aulos modes, played in *descending* direction.

† For the origin of the true tones and scales see chapter 4.

‡ It is known that a scientist of the spirit may only give indications when asked.

* Transposed by the author onto the degrees of the generic tone c^5 = 4096 Hz.

Table 16
Two indications given by Rudolf Steiner to Lewerenz

1

First indication given by Rudolf Steiner to Wilhelm Lewerenz (according to von Lange[12])

	scale of seconds	difference	cosmic scale	
a				
	A	seventh	G	
	G	sixth	E	
	F	fifth	C	
(etheric)	E	fourth	A	(astral)
	D	third	F	
	C	second	D	
	B	prime	B	
centre	E= ♃	common basis B = ☽ below the Sun		
b	scale of seconds	difference	cosmic scale	
	F	seventh	G	
	E	prime	E	
	D	seventh	C	
(etheric)	C	sixth	A	(astral)
	B	fifth	F	
	A	fourth	D	
	G	third	B	
centre	C = ♂	common basis E = ♃ (above the Sun)		

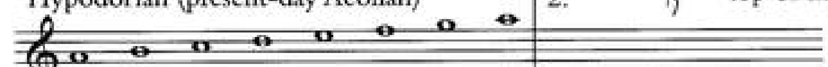
2[++]

Second, unpublished, indication given by Rudolf Steiner to Wilhelm Lewerenz

Church tones

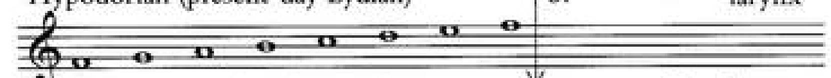
a
g
e

[+]Hypodorian (present-day Aeolian) 2. ♄ top of the head

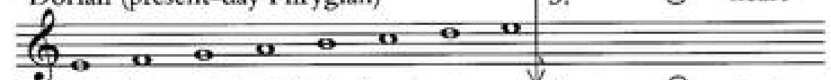


[+]Hypophrygian (present-day Mixolydian) 7. ♃ brow

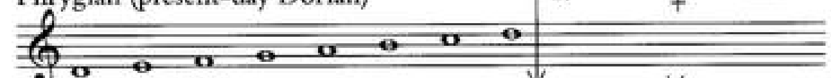
[+]Hypodorian (present-day Lydian) 5. ♂ larynx



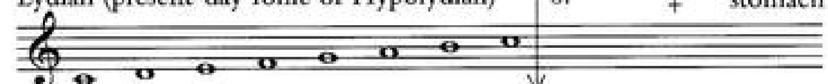
[+]Dorian (present-day Phrygian) 3. ☉ heart



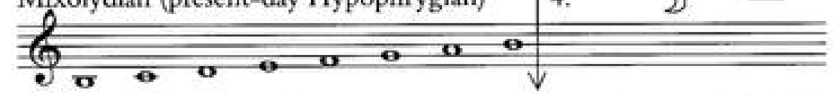
Phrygian (present-day Dorian) 1. ♀ —



Lydian (present-day Ionic or Hypolydian) 6. ☿ stomach



Mixolydian (present-day Hypophrygian) 4. ☽ —



In the late Roman era the Dorian E was no longer the norm, but the Lydian C. The Lydian C was the lowest tone. Saturn is furthest away, therefore the lowest tone, while the Moon is the highest.

[+]Main keys of the kithara harmony: a, g, e.

[++]Read from right to left.

Together, the seven descending octave scales of Indication 2 make up the great system Teleion of antiquity which progressed from the highest tone, Nete Hyperboleion, to the lowest tone, Hypate Hypaton, and was made up entirely of fifth tones.[4,5] If one stays within the area of non-altered fifth tones, the highest tone can only be an A and the lowest B. It follows from this, seemingly unequivocally, that the tones and scales of Indication 2 relate to the Apollonian music stream of antiquity, i.e., all of the tones belong to the row of twelfth tones and all intervals are true intervals. This assumption is further supported by the double naming of each scale—on the one hand the Apollonian and on the other hand the names of the church modes (see Table 8). This explains the remark 'main keys of the kithara A, G, E'. The comments at the bottom of Indication 2 undoubtedly relate to its scales. The Lydian C thus also belongs to the Apollonian true-tone stream.

Table 17

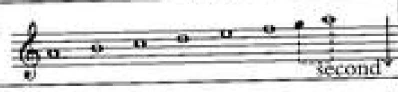
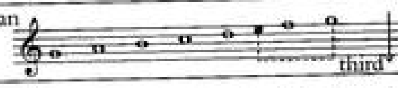
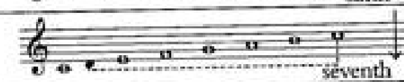
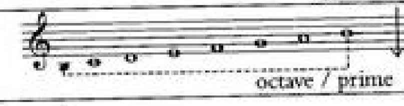
In Table 17, Indications 1a and 2 are combined. It offers a possible solution to the contradiction between the planet-tone concordances of Indication 1a and the initial tones of the scales in Indication 2. If we follow each line from right to left (i.e., against the usual direction of reading), we get the difference interval of a seventh between the Saturn tone G of the cosmic-scale sounding in the astral and the Sun tone A of the second-scale sounding in the etheric. The Sun tone A is the initial tone in the Hypodorian octave scale a^2, g^2, f^2, e^2, d^2, c^2, b^1, a^1, which is associated with the planet Saturn. This association is puzzling until one notes that it coincides with the principle of interval inversion. In the Hypodorian octave scale the Saturn tone G is on the second degree of the scale and forms an interval of a second—the inversion of the interval of a seventh—with the beginning Sun tone A. In other words, the interval difference of a seventh, which makes the transition between the Saturn tone G in the astral and the Sun tone A in the etheric, appears in the a^2–a^1 Saturn scale, sounding in the physical world, as the interval of a second in its inversion. In the next line, a descending difference interval of a sixth leads from Jupiter tone E in the astral world to Saturn tone G in the etheric world. In the earthly g^1–g^2 octave scale which descends from Saturn G and is associated with the planet Jupiter, Jupiter tone E is on the scale degree of a descending third. The third is the inversion of the sixth. Similar correspondences apply for the other five scales.

We may therefore say that the planetary tone with which a scale is associated does not come at the beginning or end of a scale but at the point in the scale which creates the inversion of the difference interval to the initial tone. Only in the Moon scale is the Moon tone B identical with the initial tone because the difference interval is the prime. In all seven cases, the cosmic scale, difference interval, second scale and the octave scale sounding in the physical are connected with one another in this way. Whether sounding in the astral, etheric or physical, each of the seven tones of the descending true-tone scale is associated with the same planet. The difference intervals go from the planetary tones of the cosmic scale sounding in the astral to the planetary tones of the second scale sounding in the etheric. The tones of the latter are the initial tones of the octave scales which sound in the physical world.

The inversion of the intervals points to the fact that the point of departure for all human activity in antiquity, including music, was not the physical, but the astral, i.e., the supersensible world. This is evident from the association of the tones with the planets, where, in the cosmic third scale, G is the highest and therefore fastest moving tone and associated with Saturn. Rudolf Steiner explained in his Cologne lectures[20] how, seen from the point of view of the astral world, Saturn was the closest and therefore moved the fastest, so that its movement corresponded with the highest tone. Steiner also specified the numerical ratios, showing how the other planets, seen from the point of view of the astral world, moved progressively slower in descending order. In the astral everything appeared in its inversion[26]—seen from the point of view of the physical world, Saturn moved the slowest and was therefore the lowest tone.

The association of the planets with the different parts of the human anatomy, as shown in Tables 16 and 17, was already made by the Greeks and they are mentioned by Rudolf Steiner in, for example,

Table 17
Comparison between Indications 1a and 2a[+]

Indication 2						Indication 1a				
Greek Names			descending octave scales	association		scale of seconds etheric		difference	cosmic scale astral	
tetrachord structure	tones[+++]	scales	physical	planet	organ	planet	tone		tone	planet
1. Dorian 2. Phrygian	Nete Hyperboleion –Mese	[++]Hypodorian		Saturn	top of the head	Sun	A	seventh	G	Saturn
1. Phrygian 2. Lydian	Paranete Hyperboleion –Lichanos Meson	[++]Hypophrygian		Jupiter	brow	Saturn	G	sixth	E	Jupiter
1. Lydian 2. Mixolydian	Trite Hyperboleion –Parhypate Meson	Hypolydian		Mars	larynx	Venus	F	fifth	C	Mars
1. Dorian 2. Dorian	Nete Diezeugmenon –Hypate Meson	[++]Dorian		Sun	heart	Jupiter	E	fourth	A	Sun
1. Phrygian 2. Phrygian	Paranete Diezeugmenon –Lichanos Hypaton	Phrygian		Venus	—	Mercury	D	third	F	Venus
1. Lydian 2. Lydian	Trite Diezeugmenon –Parhypate Hypaton	Lydian		Mercury	stomach	Mars	C	second	D	Mercury
1. Mixolydian 2. Dorian	Paramese Diezeugmenon –Hypate Hypaton	Mixolydian		Moon	—	Moon	B	prime	B	Moon

centre E = ♃ common basis B = ☽ below the Sun

[+]Pre-Christian. To be read from right to left and above downwards.

[++]Main keys of the kithara harmony: a, g, e.

[+++]Nete Hyperboleion–Hypate Hypaton: range of the great system of Teleion = one octave and a minor seventh.[4]

the Torquay lectures of 1924.[59] The significance of their appearance in Indication 1 is not known to the author.

In Indication 2, the three main keys of the antique System Teleion are stated as being the Hypodorian a^2–a^1, the Hypophrygian g^2–g^1 and the Dorian e^2–e^1.* The descending tones of the second scales already make up two connected Dorian scales: A, G, F, E/E, D, C, B, and because the Dorian fifth tone scale e^2–e^1 comes in the middle, i.e., in a central Sunlike position, the basic mood of the seven octave scales can be taken to be Dorian. For the Greeks, the Dorian tetrachord represented the Sun and thereby also the sublime Sun spirit. The descending direction of the scales may point to the fact that this sublime Sun spirit was nearing the earth but not yet present on it.[31, 60]

Table 18

At the bottom of Indication 2 in Table 16 we read the comment:

> In late Roman times, the E, the Dorian, was no longer the standard proportion, but the Lydian C. The Lydian C came to be the lowest tone. Saturn was the furthest away, therefore the lowest tone; the Moon was the highest.†

The author is not aware of any further clarifications of this point given by Rudolf Steiner. She has therefore felt free to consider a restructuring in connection with Indication 1a similarly to the way this was done with the scales of Indication 2. The planetary sequence of the scales in Indication 2 is the same as that for the tones of the cosmic scale in Indication 1—Saturn uppermost, then Jupiter, Mars, Sun, Venus, Mercury, and Moon at the bottom. This is because in late Roman times the scales were still descending scales. The Lydian C scale is therefore at the top, although as Saturn it is the lowest; the Mixolydian B scale, even though it is the highest is at the bottom, being Moon. The other five scales are arranged with the planets according to the Ptolemaic system.

The author hopes that this will not be seen as a violation of the indications and she also does not wish to deny that other explanations may also be possible. However, the inverted positions of the scales' initial tones, with the Ptolemaic planetary sequence retained, point to secrets that may lie hidden in these indications.

The comment quoted above points to the fact that, beginning with late Roman times, there must have been a restructuring of the octave scales in Indication 2. The scales in Table 18 are descending scales. However, as it is expressly stated that the Lydian C is the lowest tone—with Saturn the most distant, so that this is the lowest tone and the Moon the highest—the sequence of the scales is therefore ascending. It seems to mean that from late Roman times the point of departure for music was no longer the super-sensible astral world but it was here, on the physical earth. This explanation is further supported by the difference intervals that connect the scales.‡ The difference intervals between the second scale and the cosmic scale do not follow one another step by step as in Table 17, but by leaps of a fourth. Maybe this is an indication that the connection between astral and etheric has become looser.

An attempt will now be made to show that viewed together, the first, fourth and seventh scales in Table 18 seem to mirror the fact that relatively late Rome was the time of the Mystery of Golgotha—a connection to which Rudolf Steiner would frequently refer.

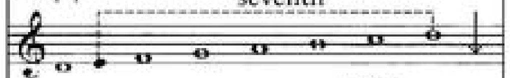
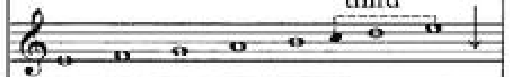
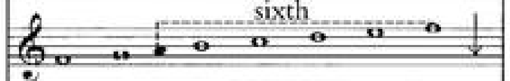
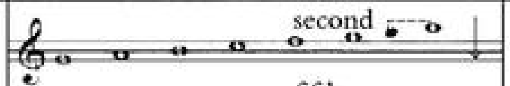
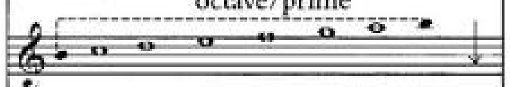
* The tetrachord structure of all seven octave scales is given to the left of Table 17.

† It is taken for granted that 'E, the Dorian' and 'the Lydian C' also mean the Dorian and Lydian scales for if one looks up the names of the Greek scales in Bindel[8] and Pfrogner,[5] for example, these are called e^1–e Dorian and c^1–c Lydian. The author has adopted this terminology for the whole of this chapter although the old names, e.g. Pythagorean Dorian octachord, are used throughout the rest of the work.

‡ As in Table 17, the second scale is based on the initial tones of the octave scales. Though only a heptachord, this second scale forms the true C major scale.

Table 18+

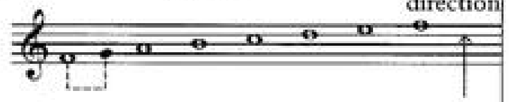
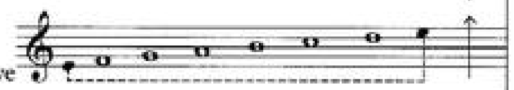
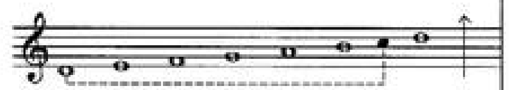
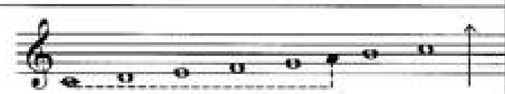
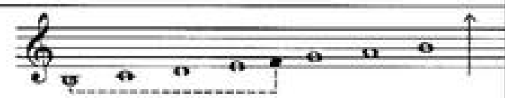
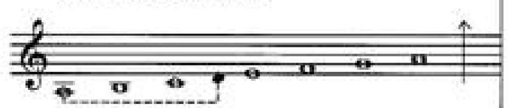
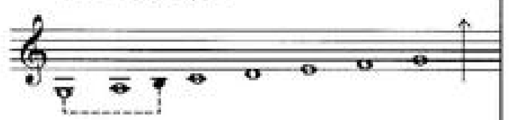
Comments at the bottom of Indication 2 supplemented and compared with Indication 1a from Table 16 (late Roman era)

Indication 2	Comments				Supplements			Indication 1a	
	descending octave scales				scale of seconds etheric		difference interval between 2 and 1a	cosmic scale astral	
Greek scale name	physical	degree interval	scale direction	planet association	planet	tone		tone	planet
Lydian	 ++	fourth	↓	Saturn	Mars	C	fifth	G	Saturn
Phrygian		seventh	↓	Jupiter	Mercury	D	second	E	Jupiter
Dorian		third	↓	Mars	Jupiter	E	sixth	C	Mars
Hypolydian		sixth	↓	Sun	Venus	F	third	A	Sun
Hypophrygian		second	↓	Venus	Saturn	G	seventh	F	Venus
Hypodorian		fifth	↓	Mercury	Sun	A	fourth	D	Mercury
Mixolydian		octave/prime	↓	Moon	Moon	B	prime	B	Moon

+Read from right to left and from above to below
++C becomes the lowest tone

Table 19⁺

Indication 1b supplemented according to Indication 2 (see Table 16)

Greek tetrachord structure	Church names according to Table 8	Supplements			No.	Indication 1b				
		ascending octave scales				scale of seconds		difference interval	cosmic scale	
		degree intervals	physical	scale direction		etheric			astral	
						planet	tone		tone	planet
2 Dorian 1 Mixolydian	Lydian	second			7	Venus	F	second	G	Saturn
2 Lydian 1 Lydian	Phrygian	prime/octave			6	Jupiter	E	prime	E	Jupiter
2 Phrygian 1 Phrygian	Dorian or Hypomixolydian	seventh			5	Mercury	D	seventh	C	Mars
2 Dorian 1 Dorian	Ionian or Hypolydian	sixth			4	Mars	C	sixth	A	Sun
2 Mixolydian 1 Lydian	Hypophrygian	fifth			3	Moon	B	fifth	F	Venus
2 Lydian 1 Phrygian	Aeolian or Hypodorian	fourth			2	Sun	A	fourth	D	Mercury
2 Phrygian 1 Dorian	Mixolydian	third			1	Saturn	G	third	B	Moon
						centre = Mars, common basis E = Jupiter above the Sun				

⁺Christian era. To be read from left to right and from below to above.

planetary incarnations—G = Saturn, A = Sun, B = Moon, C = Mars, D = Mercury, E = Jupiter, F = Venus.[31] Confirmation that it is right to read 1b in ascending direction is given by the fact that the second scale then forms two connected Dorian (Sun) tetrachords—G, A, B, C, D, E, F—as the descending row of the second scale in 1a also does—A, G, F, E, D, C, B. If one moves on to the cosmic scale and reads this in ascending direction, this will give the sequence of the post-Atlantean periods. These extend from the ancient Indian period, which was mainly under the spiritual influence of the Moon, to the seventh, future period which will probably be connected with Saturn.[48]

Reading the scales in ascending direction gives a still further and equally magnificent insight. According to Rudolf Steiner, the insights people had in the ancient Indian period recapitulated, under the influence of the Moon, the first Polaric age of Earth evolution which in turn recapitulated the old Saturn evolution.[60, 61] This is indicated by the juxtaposition of the Moon tone B of the cosmic scale and the Saturn tone G of the second scale. In the same way, the mystery wisdom of the Persian age, which was under the influence of Mercury, recapitulated the second Hyperborean age of Earth evolution, which in turn was a recapitulation of old Sun evolution. Accordingly the Mercury tone D in the cosmic scale is opposite to the Sun tone A of the second scale. The third, Egypto-Chaldean period of civilization, mainly under the influence of Venus, recapitulated the third, Lemurian period of the Earth, where knowledge and insight was concerned, and this was a recapitulation of old Moon evolution; the Venus tone F in the cosmic scale is thus opposite to the Moon tone B in the second-scale. The fourth, Graeco-Latin period, mainly under the influence of the Sun, recapitulated the Atlantean age. This Atlantean age was the first Earth period that was not a recapitulation of an earlier stage of planetary evolution and therefore belonged in the fullest sense to the Earth itself. The Sun tone A of the cosmic scale is opposite to the Mars tone C of the second scale. Mars represents the first half of Earth evolution.

With the resurrection of the Christ, the second, Mercury half of Earth evolution began. The fifth post-Atlantean civilization began in AD 1413 and is mainly under the influence of the planet Mars.[48] The Mercury tone D in the cosmic scale is thus opposite to the Mars tone C of the second scale. The sixth post-Atlantean period will be under the influence of Jupiter and will point to the future Jupiter evolution of Earth. The Jupiter tone E is therefore present in both scales. Finally the seventh post-Atlantean period may be under the influence of Saturn and prophetically point to the next but one evolution of Earth, the Venus incarnation. This explains why the Saturn tone G of the cosmic scale is opposite the Venus tone F. While the tones of the second scale reflect the planetary incarnations of Earth, the tones of the cosmic scales give insight into the development of human conscious awareness on Earth throughout the seven post-Atlantean periods and show how these relate to the great evolutionary stages of Earth.

The seven descending scales of Indication 2 in Table 17 and the seven ascending scales in Table 19 both have the same tetrachord structures but run in opposite directions, the former descending and the latter ascending. More important than the change in direction is that the tetrachord structures of the 1st, 2nd, 3rd, 5th, 6th and 7th scales swap positions reciprocally in relation to the middle, 4th, scale. The numbers 1–7 show this in Table 20 (see p. 100).*

The qualities of the scales go through a complete inversion because the change in direction results in a change in the sequence of the intervals that form the tetrachords so that the tetrachords themselves are no longer the same. Whether the sequence of intervals forming a tetrachord ascends or descends does not make any difference, however, to the quality of a tetrachord. The character of the Dorian tetrachord is still courageous, the Phrygian painful, tense and severe, the Lydian devoted and the Mixolydian over-tensed but also self-sacrificing. Each scale has two tetrachords and the order greatly determines the overall character, but a detailed description of each scale would far exceed the present work. Three scales contain Dorian tetrachords. The middle scale in descending direction is the Dorian Sun scale. In

* The reading of this table is greatly helped by comparing it directly with Tables 17 and 19. (Translator's note.)

Table 20
Comparison between Tables 18 and 19

Table 18							Table 19		
Great System Teleion				Tetrachords					
Greek scales names	descending		order of the scales	first		second	order of the scales	ascending	church scale names
Hypodorian	a^2–a^1	♄	1	Dorian –		– Dorian	7	f^1–f^2	Lydian
Hypophrygian	g^2–g^1	♃	2	Phrygian –		– Lydian	6	e^1–e^2	Phrygian
Hypolydian	f^2–f^1	♂	3	Lydian –		– Phrygian	5	d^1–d^2	Dorian or Hypomixolydian
Dorian	e^2–e^1	☉	4	Dorian –		– Dorian	4	c^1–c^2	Ionian or Hypolydian
Phrygian	d^2–d^1	♀	5	Phrygian –		– Mixolydian	3	b^1–b^2	Hypophrygian
Lydian	c^2–c^1	☿	6	Lydian –		– Lydian	2	a–a^1	Aeolian or Hypodorian
Mixolydian	b^1–b	☾	7	Mixolydian –		– Phrygian	1	g–g^1	Mixolydian

ascending direction it is the true-tone C major scale. The first of the ascending scales and the seventh descending scale end with a Dorian tetrachord. When one compares the movement of both groups, beginning with the middle scale, the following may result.

In antiquity the movement went from the middle scale e^2–e^1 down to the dominant scale b^1–b (2nd tetrachord is Dorian), made a great sweep to the sub-dominant scale a^2–a^1 (1st tetrachord is Dorian) at the top, and descended from there again to the middle. In this movement the music is held in continual suspension. The movement of the ascending scales of today is up from the middle C major scale to the sub-dominant f^1–f^2 (2nd tetrachord is Dorian), makes a great sweep to the dominant g–g^1 (1st tetrachord is Dorian) at the bottom and ascends from there to return to the tonic. This movement is musically filled out and describes the fundamental structure of modern harmony: tonic, sub-dominant, dominant, tonic, the inversion of the great antique System Teleion which commenced at the time of the Mystery of Golgotha.

The decisive difference in mood between the two directions can be experienced by comparing the 'Hymn of Apollo' from *c.* 150 BC with the C major prelude from the first book of *The Well-tempered Clavier* by Bach. The difference is very clear even when the Bach is heard in equal-tempered tuning. It is even clearer, though, if one plays both compositions on a lyre tuned to the scale of twelve fifths, for the beauty and liberating strength of the latter is still more convincing.

A new development in music could only come in about the tenth century,[17,62] when the old descending scales were transformed into ascending scales in musical practice and the Greek Phrygian scale was elevated, as the church Dorian scale, to the central scale (see Table 8). The latter consists of two Phrygian tetrachords, with the semitone step in the middle, flanked by two major seconds. The symmetry of this tetrachord makes it possible to play the scale in either ascending or descending direction without changing the tetrachord structure, making it possible for music practice to change from descending to ascending in scale direction. The Phrygian tetrachord corresponds with the present minor. This occurs twice in the church Dorian scale, which gives it an incredibly contracting, even painful effect. In Table 19 this scale is next to D in the second scale and next to C in the cosmic scale, in other words, in the fifth post-Atlantean period of civilization. This painful feeling of contraction with its structure points to the time in which we are, when the independent human being is focused completely to the minute point of the 'I' for the development of the spiritual soul. This was prepared for in the Middle Ages when the D–d scale was made the central one. This alone will not allow humanity to progress. The human 'I' can only move ahead in the development of the spiritual soul if it takes up the Christ impulse. The D in the second scale being opposite the C in the cosmic scale, the fundamental of the C major true tone resurrection scale, which gained in importance from the twelfth to the sixteenth centuries, points to this.[62,68] First of all this scale, connected with the Mystery of Golgotha, and its Dionysian sister, the C major just scale arising from the changed aulos Mars mode (chapter 9, pp. 41–2) led to a further development in occidental music. How these two scales moved from the fourth into the fifth post-Atlantean epoch and how they formed the triads singular to the degrees of each scale during this change will be shown in the next chapter, using the example of Rudolf Steiner's tone spiral.

Hymn to Apollo
c. 150 BC

Hear, O white-armed daughter of Zeus, in Helikon's dense forest, the father's deep thunder. Hurry thither, to celebrate brother Phoebus of the golden hair in song, who visited, above the high ragged cliffs of Parnassus with the illustrious daughters of Delphi the wonderful waters of Kastali's spring, in the heights of Delphi, the mighty god of the Oracle.

golden hair in song, who visited, above the high ragged cliffs of Parnassus with the illustrious daughters of Delphi the wonderful waters of Kastali's spring, in the heights of Delphi, the mighty god of the Oracle.

Bach Prelude [64]

20 Rudolf Steiner's Tone Spiral

On 23 August 1915, Rudolf Steiner gave two tone spirals to the eurythmists working in Dornach. Although the tone spirals were given for eurythmy (i.e., the moving human being making music visible in space) and are therefore subject to the laws that underlie this art, they can throw a lot of light on purely musical questions and one of them may have a connection with our fifth post-Atlantean cultural period.

The tone spirals consist of the ascending C major scale, with its single tones laid out degree by degree, the one on three twelvefold divided circles, the other in a spiral. The tone spirals, which are not to be found in Rudolf Steiner's printed lectures, were first printed in 1965 based on handwritten notes and oral tradition.[18] Different versions exist as a result, spiralling inwards and outwards. Because of this the author may be permitted to refer to the tone spiral handed down to her by her mother Lucy Neuscheller.* This is shown in Fig. 1. It consists of five complete octaves of the C major scale and is therefore the same as the one printed by Elena Zuccoli in her interesting book on tone eurythmy work in the first eurythmy school in Stuttgart,[65] and has come down from H. Hollenbach. Its register is likely to be from C to c^4, a range which Rudolf Steiner often spoke of.[30] In the original drawing handed down by Lucy Neuscheller, the beginning and end were indicated by a dividing line. To make it easier to understand, the author has underlined the initial tone C, added arrows and labels and numbered the twelve points of the circle in ascending order. She has also marked correspondence with the signs of the zodiac, the arrangement of which is the same as that given by Rudolf Steiner for the eurythmic choreography of his Twelve Moods. The resulting connection between the triads and the zodiac was confirmed by eurythmist Friedel Thomas-Simons in a conversation with the author in 1963.† These triads need to be read from the circumference towards the middle and are made up of intervals of a descending octave plus a sixth and therefore represent cosmic rather than earthly triads.

According to this arrangement C and C major begin in Aries and move from there in spiralling fashion through five octaves to the sixth and highest c in the sign of Pisces. This may point to a connection with the fourth and fifth post-Atlantean periods, as the spring equinox of the Sun was in Aries during the fourth post-Atlantean period, and is in the sign of Pisces in the fifth. Both the Apollonian C major resurrection scale and the just C major scale which is derived from the Dionysian Mars mode (see chapter 8 and chapter 9, pp. 41–2) may come into consideration in this tone spiral. The spiral shows how both of these musical streams and their common fundamental arrived in the modern age.

From the anthroposophical point of view, the human being is created out of the powers of the zodiac.[32] The tone spiral and the signs of the zodiac in Fig. 1 show C moving through five octaves from Aries (head) through Scorpio (reproductive organs), Gemini (symmetry), Capricorn (knee), Leo (heart) to Pisces (feet), and we can see how this tone is anchored in the human being. Further points of view

* Johann Leopold (1885–1976) and Lucy (1888–1962) Neuscheller-van der Pals heard Rudolf Steiner lecture in Berlin and Helsinki. They became members of the Anthroposophical Society as early as 1913. In 1915 they came to Dornach, Switzerland, to help with the wood carving for the First Goetheanum. Lucy soon became keenly interested in eurythmy and was part of the Dornach ensemble until 1923. During this time she, together with other eurythmists, received the tone spiral from Rudolf Steiner and consequently applied it in her work. For the staging of the Knight's Hall scene in Goethe's *Faust*, II, Act 1, Marie Steiner asked her to perform the role of Helena. She was one of the eurythmists who took part in the memorable last performance given in the First Goetheanum before it burned down. In 1923 the call for a eurythmist came from America, and Rudolf Steiner encouraged Lucy and Leo to answer it. Lucy introduced eurythmy and Leo speech formation into the USA. Both were representatives of these arts for several decades. In 1923 they founded the 'Goetheanum School of Eurythmy' in New York, which was endorsed by Rudolf Steiner, trained students and gave performances in many parts of that large country. Being a conscientious artist Lucy Neuscheller gave the tone spiral to the author shortly before she died.

† Friedel Thomas (1896–1974) was part of the eurythmy stage group at the Goetheanum under Rudolf Steiner, together with my mother.

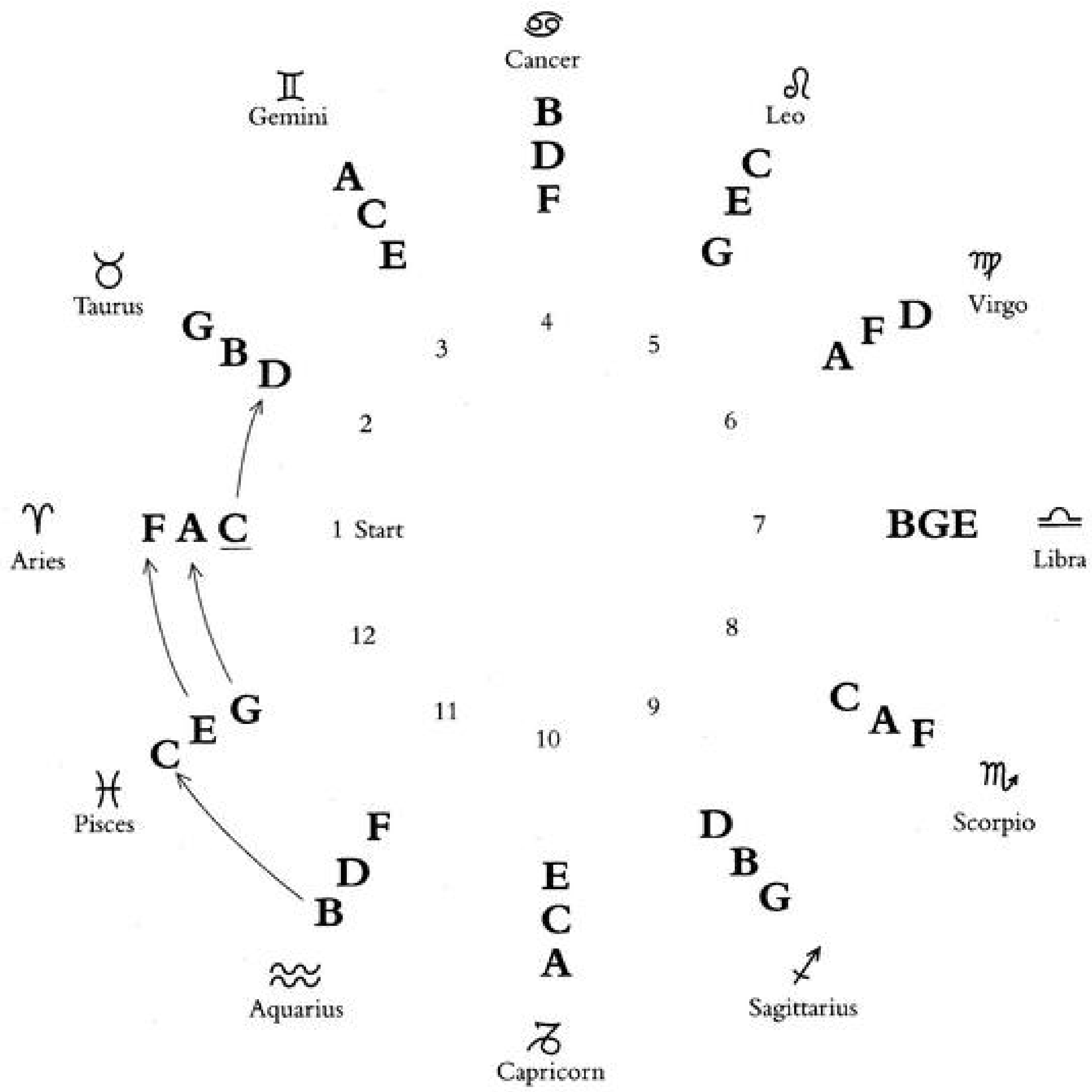


Fig. 1 Rudolf Steiner's tone spiral according to Lucy Neuscheller, with added zodiac signs

are thus added to those given in chapter 17, showing how the tone c = 128 Hz and its octaves belong to the human being.

A serious problem needs to be considered here. It is important how Rudolf Steiner gave the tone spiral in 1925. Also it is known that he was in the habit of making the work of his students generally known. In 1922 Hermann Beckh published a book on the spiritual nature of the keys,[88] speaking of the importance of C and C major and their connection with Aries for the present fifth post-Atlantean period. A copy of this work with a dedication to Rudolf Steiner is contained in the latter's library. However, repeated searches have shown that Rudolf Steiner made not even a single reference to Beckh's work in his own works and lectures. Beckh's book on Egypt was mentioned several times, on the other hand. Could this be because Rudolf Steiner considered the tone C and C major to be in the region of Pisces today and no longer in Aries?

Before we go on to the next chapter where we will discuss a further indication given by Rudolf Steiner, namely, 'C is always the prime', a historical fact will be considered that had a major effect on the further development of Western music.

At the beginning of modern times the experience of the third gave rise to the development of polyphony and harmony,[30] and modern science, progressively limiting attention to the physical world, also began to develop. The result was a preference for the peaceful sound of the beat-free just third. The development of the Apollonian true-tone scale, which could have made its appearance, was delayed (see Grammateus, chapter 13). The altered Dionysian Mars mode on the fundamental C became the fundamental scale for the whole system of music, its structure applied to all other scales, and the circle of fifths evolved. The inevitable outcome was the harmonically unbearable acute fourth and grave fifth,[13,14] which one sought to avoid by continually altering the pitches of the scale degrees (see chapter 7, pp. 23–5). Awareness for singular qualities of tones was largely lost. On string and wind instruments, which were gradually brought together to be the orchestra, these changes were easy to execute. With keyboard instruments it was necessary, however, to falsify the intervals by means of temperament, which dulled people's experience of the intervals. In spite of this, great and mighty works of art, capable of moving human beings deeply, were able to arise from this development because the inheritance of an earlier, spiritually orientated musicality was still alive in the composers of the eighteenth and nineteenth centuries. It was only when this inheritance had almost completely disappeared that the twelve equal-tempered minor seconds were treated as equally important musical units, e.g. by Schoenberg. One needs to bear in mind that eight deliberately falsified, i.e., untrue intervals (see chapter 10) that are definitely experienced as such by the ear, were thus given the same importance as the aurally genuine intervals. From about the same time, a technology has developed which is increasingly more able to simulate the musical experience with electrical reproductions in the form of records, tapes, radio, amplification and later electronic imitation tones. Our aural experiments presented in chapter 15 show that electrical reproduction extinguishes the inherent singular quality of the tones, replacing it with its own. This becomes comprehensible when one considers how human beings imitate the qualitative nature of tones they perceive, as we know from Rudolf Steiner.[45] Listening to electrically reproduced tones, therefore, human beings also imitate the qualitative nature of these tones—a 'grinning vacuum' that erodes soul and spirit.

This destructive development was necessary for the attainment of human freedom. It was necessary to lose the old instinctive and compulsive way of perception in order to win through to a new experience and handling of music that is in accord with the spirit. The Apollonian true-tone C major scale on the fundamental c = 128 Hz and the seven original Dionysian aulos modes on the generic tone c^5 = 4096 Hz make it possible to connect the Christian impulse in music with genuine intervals and tones. This clears the way for a further development in diatonicism and chromaticism. This has already been considered earlier and will be taken further in the next chapter.

21 'C is Always the Prime'

The exact circumstances under which Rudolf Steiner said that 'C is always the prime' were related to the author by Rie Lewerenz as follows. During a rehearsal in the early 1920s, Rudolf Steiner said to her and two other eurythmists that C was always the prime. One of the three strongly challenged this, saying that the tonic of each scale is its prime. But Rudolf Steiner repeated: 'No, C is always the prime.'

At first glance, this assertion, as the eurythmist tried to point out, is not only at odds with the accepted view of the structure of our scales but also with the physical fact that each tone, no matter what its pitch, is the prime or tonic of the overtone and undertone rows belonging to it. This fact was well known to Rudolf Steiner, as he clearly stated in 1924 that any tone could be the fundamental, but even then he chose the tone C as the tonic in what he said.[33] His remark that C is always the prime must therefore point to profounder and more comprehensive situations.

Justifications for the tone c = 128 Hz and its octaves as common prime

We have seen that c = 128 Hz and its octaves is rooted in the human being, is linked to the Michaelic meteoric-iron Sun-substance and to the Mystery of Golgotha. This shows that C is not just any old tone. It has proved itself to be the tone which Rudolf Steiner called 'the actual C',[30] which, when recognized by Christian musicians, would be taken up as the basis for all music making, with all other tones based on it. A sense for the singularity of this tone in Christian music must have led musicians at the beginning of the modern age to use C major as a model for the structure of all major scales in our 'circle of fifths'. As the just major scale was used, the following discussion will primarily be concerned with just major scales and their relative minor scales. The just scales necessitated that the pitch of their tones changed continually (see chapter 7), which caused the sense for the true pitch of the tone C to diminish. Our aural experiments and chapters 17, 18 and 19 have, however, served the purpose of unequivocally establishing the pitch of this tone. The tone C, designated 'the common prime' by Rudolf Steiner, can therefore only be the tone c = 128 Hz which he gave to Schlesinger (chapter 17) and on which we also wish to base ourselves.

c = 128 Hz is the prime of all just scales in the Dionysian circle of fifths

For objective proof that C really is the prime of all just scales in the circle of fifths, we must first of all list the tonics of these scales as they proceed from C. The tonics of the 14* just major scales are C, G, D A, E, B, F♯, C♯, G♭, D♭, A♭, E♭, B♭ and F. The tonics of the 14 parallel minor scales, as submediants of the major scales, form a descending 6:5 just minor third from the major scales' tonic. This means that when a just scale begins on a *true* tone, the parallel just major or minor scale begins on a *just* tone and contains the scale degrees appropriate to it; likewise, when a just scale begins on a *just* tone, the parallel just major or minor scale begins on a *true* tone and contains the scale degrees appropriate to it. (See chapter 7.)

It follows that each individual scale of the 28 scales in the circle of fifths must of necessity have two possible tonics. These 56 (2 × 28) tonics are shown in Table 21.

The Hz show that there is no doubling-up of just tonics.† However, in the case of the true tones there is. Therefore eleven tones can be subtracted, provisionally leaving 45 tonics.

The tonics of the major scales make pairs of ascending and descending intervals from the tone C: F and G a fifth, B♭ and D a major second, E♭ and A a minor third etc. The true tonics of the major scales make true intervals and the just tonics just intervals with C.

* Actually 15, but C♭ is not mentioned because it is as good as never used.

† E.g. just E♭ and just A: just E♭ as 6th degree 5:3 in true G♭ major sounds in the middle octave at 299.662 Hz, while just E♭ as 3rd degree 6:5 in true C minor sounds at 307.200 Hz; just A as 6th degree in true C major sounds at 213.333 Hz, and just A as 3rd degree in true F♯ minor sounds at 218.699 Hz.

Table 21
List of scale tonics

	Hz	Just major scale	Interval to C		Interval to C	Relative just minor scale	Hz	
Descending intervals	89.898	true G flat	augmented 4th		major sixth	just E flat	76.800	Descending intervals
	91.022	just G flat				true E flat	75.852	
	67.424	true D flat	major 7th		whole tone	just B flat	115.200	
	68.267	just D flat				true B flat	113.778	
	101.136	true A flat	major 3rd		fifth	just F	86.400	
	102.400	just A flat				true F	85.334	
	75.852	true E flat	major 6th		**prime**	just C	129.600	
	76.800	just E flat				**true C**	**128.000**	
	113.778	true B flat	whole tone		fifth	just G	189.630	Ascending intervals
	115.200	just B flat				true G	192.000	
	85.334	true F	fifth		whole tone	just D	142.222	
	86.400	just F				true D	144.000	
	128.000	**true C**	**prime**	**= 128 Hz**	major 6th	just A	213.333	
Ascending intervals	129.600	just C				true A	216.000	
	192.000	true G	fifth		major 3rd	just E	160.000	
	194.400	just G				true E	162.000	
	144.000	true D	whole tone		major 7th	just B	240.000	
	145.800	just D				true B	243.000	
	216.000	true A	major 6th		augmented 4th	just F sharp	180.000	
	218.699	just A				true F sharp	182.250	
	162.000	true E	major 3rd		augmented 8ve	just C sharp	276.792	
	164.025	just E				true C sharp	273.374	
	243.000	true B	major 7th		augmented 5th	just G sharp	202.500	
	246.037	just B				true G sharp	205.031	
	182.250	true F sharp	augmented 4th		augmented 2nd	just D sharp	151.875	
	184.528	just F sharp				true D sharp	153.773	
	273.374	true C sharp	augmented 8ve		augmented 6th	just A sharp	227.812	
	276.792	just C sharp				true A sharp	230.660	

All just major scales have the same internal interval structure as the just major scale beginning on true C, but each scale has its own particular character. The interval structure is therefore not character forming. This is given by the character of the tonic which in turn is influenced by the character of the interval between the tonic and the common prime C.* If we thus compare the just major scales beginning on true A and true E, the former has an effect like Sun-saturated light streaming out, while the latter shines out as though from within. The tone true A has breadth and light† and makes a 27:16 true major sixth with C, an interval that opens out more than the fifth, while true E makes a 81:64 true major third with C, an interval that lives and shines within the human being.[33]

In the parallel minor scales the characteristic quality of the interval between the tonic and C is also clearly apparent. Thus the tonic of just F minor makes a fifth with true C. Rudolf Steiner said that the

* It is hardly possible to find more appropriate descriptions of the intervals that truly reflect the real situation than those given by Rudolf Steiner.[20, 23, 33] We will therefore base ourselves on his descriptions.

† See aural experiments in chapter 16.

fifth was the human being.[33] The C:F fifth is descending, i.e., getting denser, and 'minor' means going inwards. The passionate character of this key is therefore due to the descending fifth. On the other hand B minor makes an ascending major seventh with C. This is the interval of which Rudolf Steiner said that one goes out of oneself.[33] Being a minor key, this going out of oneself is expressed in a very inward way. Bach's great mass, a rite that connects the human soul with the world of the spirit, begins in this key.

These few examples may suffice to show that the interval between the first tone of the scale and C plays a major part in determining the character of a key. This shows a connection between the tonic's character and the character of the interval which it makes with true C. A first connection becomes apparent between the tonics of the scales in the circle of fifths and C as a common prime. This is not enough, however, to show that C is really the prime of all keys. It has been shown that the 45 named tonics relate directly to the overtone, undertone or true-tone row of c = 128 Hz. This will be shown below, beginning with an examination of the connection between the degrees of the scales to the overtone and undertone rows of their tonics and to C.

The just tonics are simultaneously the mediant, submediant or leading note degrees of the just scales which begin on true tonics. We can find the relationship of just tones to true C when we discover the relationship between the mediant, submediant and leading note degrees of the scales to the tonic. We have already seen in chapter 7 that the submediant of the just major scale is the fifth overtone of the subdominant and therefore foreign to the scale's tonic. With the just minor scales the situation is more complex. The submediant of the just minor scale is the fifth undertone of the tonic, but neither the mediant nor the leading note degrees are contained in its overtone or undertone rows. The mediant is the fifth undertone of the dominant and the leading note is the fifth undertone of the supertonic. This can be illustrated with the mediant and leading note of the just C minor scale.

Proportion between tonic and mediant in the minor

tonic = c = 128 Hz   3:2   dominant = g = 192 Hz;
2 octaves higher g^2 = 768 Hz; 768 Hz ÷ 5 = 153.667 Hz;
153.667 Hz = just E♭ = submediant 6:5 of the diatonic just C minor scale.

Proportion between tonic and leading note in the minor

tonic = c = 128 Hz   9:8   supertonic = d = 144 Hz;
3 octaves higher d^3 = 1152 Hz; 1152 Hz ÷ 5 = 230.400 Hz;
230.400 Hz = just B♭ = leading note 9:5 of the diatonic just C minor scale.

The above proportions hold good for all just minor scales. It is not commonly known that the different just tones of the above scales are nevertheless subject to a law. In the just major scales each new just tone in the circle of fifths is always the 15th overtone of the true tonic. In the just major scale the 15th overtone of the true tonic always is the leading note: tonic true f = 170.667 Hz × 15/8 = leading note just e^1 = 320 Hz. In the major scale of the next higher true tone in the circle of fifths (true C), just E becomes the fifth overtone, i.e., the mediant: tonic true c^1 = 256 Hz × 5/4 = mediant just e^1 = 320 Hz. In the major scale beginning on the true tone two fifths higher in the circle of fifths (true G), just E is the submediant but no longer appears in the overtone row of the scale's tonic: tonic true g = 192 Hz × 5/3 = submediant just e^1 = 320 Hz. (See chapter 7.)

The relationships between just tones in the just minor scales with true-tone tonics is again more complicated. Here each new just tone in the circle of fifths always appears on the 1/15th undertone of the scale's true tonic but is not used in its diatonic just minor scale: tonic true c^1 = 256 Hz × 8/15 = just d♭ = 136.533 Hz. In the undertone row of the next lower true tone in the circle of fifths (true F), just D♭ is the 1/5th undertone, i.e., the submediant in the diatonic just scale of true F: tonic true F = 85.333 Hz × 8/5 = submediant just D♭ = 136.533 Hz. In the minor scale beginning on the true tone two fifths lower in the circle of fifths (true B♭), just D♭ is the mediant: tonic true b♭ = 227.555 Hz × 6/5 = mediant just $d\flat^1$ = 273.066, and in the minor scale beginning on the true tone three fifths lower in the circle of fifths (true E♭), just D♭ is the leading note: tonic true e♭ = 151.704 Hz × 9/5 = leading note just $d\flat^1$ = 273.066 Hz. However, true E♭ is not contained in the undertone rows of the last two tonics. So the just tones appear in and disappear from the overtone rows of the true tonics of the major and minor scales which arise from the Dionysian circle of fifths.

All just tones that are 1/15th or 1/5th undertone degrees of the lower fifths of c = 128 Hz and all just tones that are 5th and 15th overtones of the higher fifths of c = 128 Hz are also contained in the undertone and overtone rows of this c. On the other hand neither the 1/15th and 1/5th undertones of the higher fifths nor the 5th and 15th overtones of the lower fifths are to be found in the overtone or undertone rows of c = 128 Hz. Only about half of the tones contained in all the just major and minor scales are thus to be found in the overtone and undertone rows of C. This means that proof that C is the common prime cannot be established from the connections between the degrees of the scale and the just-tone row.

The solution is only found from facts and laws that have been known for a long time and were partly tabulated by Ellis,[14] but have hardly been taken into account. In chapter 4, it was shown, when considering the tritones, that the interval of the augmented just 4th between true C and just F♯ is only two cents bigger than the diminished true 5th between true C and true G♭, and that the interval of the diminished just 5th is only 2 cents smaller than the true augmented 4th between true C and true F♯. The intervals were investigated from the point of view of the musical ear and it was said that the resolution of the true forms of these dissonant intervals needs to proceed inwards because the ear demands this of tritones that are less than half an octave; any that are greater than the middle of the octave need to be resolved outwards.

We can also compare the pitch of the tones that make up the intervals with one another:

true g♭ = 179.797 Hz and just f♯ = 180.000 Hz,*
true f♯ = 182.250 Hz and just g♭ = 182.004 Hz.

The difference between the two pairs of tones is therefore roughly 0.2 Hz. In chapters 14 to 17 it was shown that tones in the neighbourhood of c that are less than 0.5 Hz apart and tones in the neighbourhood of c^1 that are 1 Hz apart prove to have the same inherent qualities—in other words, they are variants of the same pitch. True G♭ and just F♯ therefore need to be called one single tone and true F♯ and just G♭ one other single tone.

The difference in pitch between all other neighbouring just and true tones in the circle of fifths can be compared with each other in the same way as above. This is done in Table 22 where the result is the same everywhere—just and true tones of similar Hz prove to be aurally one and the same tone.

This amazing fact offers a solution to the problem at hand. With all just tonics of the major and minor scales of the circle of fifths being at the same time true tones of the true-tone row of C, each of them is in a specific interval relationship to C. All possible tonics of the major and minor scales can therefore be related by interval to C as a common prime. These intervals are partly responsible for creating the character of each single scale. We thus have proof that Rudolf Steiner's comment that C is always the prime is valid for all tones in the circle of fifths.

To help readers with the reading of Table 22, it needs to be said that it is an attempt to sum up the numerical basis of this chapter. Beginning with c = 128 Hz, the tones are in ascending chromatic order and it is shown whether they are true or just. Each related true and just tone and their Hz are placed together and separated from the next group by a line. Each tone has one line. The degree numbers of the higher fifths of C are to the right of the tones column, and those of the lower fifths to the left, the ascending interval names and cents which the tones make with c = 128 Hz are to the right, and the

* The other just F♯ = 184.528 Hz can be compared with true E𝄪 = 184.736 Hz (see Table 22).

Table 22

The tone and interval proportions of the beginning tones of the scales in the circle of fifths to c = 128 Hz and c¹ = 256 Hz as common prime. Ordered chromatically

Undertone proportion*	Interval to c¹ = 256 Hz: cents	Interval to c¹ = 256 Hz: name	Lower fifth of C	Tones: name	Tones: Hz	Higher fifth of C	Interval to c = 128 Hz: name	Interval to c = 128 Hz: cents	Overtone proportion	Degrees of the major, minor scales and aulos modes
	1200	octave		**t. C**	**128.000**		prime	0.000		t. C +&– I, t. B +&– II, t. G +&– IV, t. F +&–V, t. C♯ = j. D♭VII, j. a = t. b♭ III, j. e = t. f♭VI. 16/16 mese of all aulos modes of the generic tone c^5 = 4096 Hz.
	1198.046	j. aug. seventh		**j. B♯**	**128.144**		j. dim. second	1.954	t. C♯ 15 t. G♯ 5	
	1176.540	t. aug. seventh 1048576:531441		**t. B♯**	**129.746**	12	t. comma 531441:524288	23.460		j. C = t– B♯ I, j. B♭ = t. A♯ II, j. G = t. F𝄪 IV, j. f = t. E♯V, j. C♯ = t. B𝄪VII, t. e VI, t. a III, t. d VII.
t. B 1/15 t. E 1/5	1178.494	s. j. octave 160:81		**j. C**	**129.600**		didymus comma 81:80	21.506		
	1109.774	t. ma. seventh 243:128	5	**t. D♭**	**134.848**		t. limma 256:243	90.225		t. D♭ = j. C♯ +&– I, t. C♭ = j. B II, t. B +&– II, t. A♭ = j. G♯ +&– IV, t. G♭ = j. F♯ +&–V, t. D VII, t. A III, t. E VI, t. d♭♭ = j. c♭VII, t. cb♭ = j. b♭ III, t. g♭♭ = j. f VI.
	1107.821	j. dim. octave 256:135		**l. j. C♯**	**135.000**		j. limma 135.243	92.179	t. C 135 t. D 15 t. A 5	
t. C 1/15 t. F 1/5	1088.269	j. ma. seventh 243:128		**j. D♭**	**136.533**		j. semitone 16:15	111.731		t. D♭ = j. C♯ +&– I, t. C♭ +&– I, t. B +&– II, j. A♭ = t. G♯ IV, j. G♭ = t. F♯ V, j. D = t. C𝄪 VII, j. A = t. G𝄪 III, j. E = t. D𝄪 VI, t. f VI, t. b = j. a♯ III, t. c♭ = j. d♯ VII. 15/16 on the generic tone c^5 = 4096 Hz.
	1086.315	t. dim. octave 4096:2187		**t. C♯**	**136.687**	7	t. apotomé 2187:2048	113.685		
t. B♯ 1/15 t. E♯ 1/5	1064.809	s. j. dim. octave		**h. j. C♯**	**138.396**		b. j. limma	135.191		j. C♯ = t. e♭♭ I, j. c = t. d♭♭ II, j. F♯ = t. E𝄪V, j. b♭ = t. a♯ III, j. e♭ = t. d♯VII.
	1062.855	t. dble aug. seventh		**t. B𝄪**	**138.552**	19	t. aug. comma	137.145		
	1019.550	t. aug. sixth 9.0102:5	10	**t. E♭♭**	**142.060**		t. dim. third 10:9.0102	180.450		j. d = t. e♭♭ I, j. c = t. d♭♭ II, j. a = t. b♭♭ IV, j. g = t. a♭♭ V, t. E♭ = j. D♯ VII, t. B♭ = j. A♯ III, t. f VI.
	1017.596	j. mi. seventh 9:5		**j. D**	**142.222**		j. whole tone 10:9	182.404	t. E♭ 15 t. B♭ 5	
t. D♭ 1/15 t. G♭1/5	998.044	j. aug. sixth		**j. E♭♭**	**143.838**		j. dim. third	201.956		t. D +&– I, t. C +&– II, t. A +&– IV, t. G +&–V, j. E♭ = t. D♯ VII, j. B♭ = t. A♯ III, j. F = t. E♯ VI, j. e = t. f♭ VII, j. b = t. c♭ III, j. f♯ = t. g♭ VI. 16/18 = mese of the Jupiter mode on the fundamental c = 128 Hz.
	996.090	t. mi. seventh 16:9		**t. D**	**144.000**	2	t. whole tone 9:8	203.910		
	994.136	j. dble dim. octave		**j. C𝄪**	**144.162**		j. aug. limma	205.864	t. D♯ 15 t. A♯ 5	
t. C♯ 1/15 t. F♯1/5	974.584	s. j. mi. seventh		**h. j. D**	**145.800**		b. j. whole tone	225.416		j. D = t. C𝄪 I, j. C = t. B♯ II, j. A = t. G𝄪 IV, j. G = t. F♯V, t. b III, t. f♯ = j. g♭VI, t. e VII. 14/16 on the generic tone c^5 = 4096 Hz.
	972.630	t. dble dim. octave		**t. C𝄪**	**145.965**	14	t. dble aug. prime	227.370		
t. C 1/14	968.827	v.s. j. mi. seventh		**v.h. j. D**	**146.300**		v.b. j. whole tone 8:7	231.173		

(Continued)

Table 22 (Cont.)

Undertone proportion*	Interval to c^1 = 256 Hz: cents	name	Lower fifth of C	Tones: name	Hz	Higher fifth of C	Interval to c = 128 Hz: name	cents	Overtone proportion	Degrees of the major, minor scales and aulos modes
	929.325	t. dble aug. fifth	15	**t. F♭♭**	**149.670**		t. dble dim. fourth	270.675		j. c♭ = t. f♭♭ I, j. b♭♭ = t. c♭♭ IV, j. G♭ = t. F♯ VI.
	927.371	b. j. ma. sixth		**l. j. E♭**	**149.831**		s. j. mi. third	272.629	t. F♭ 15 t. C♭ 5	
	905.865	t. ma. sixth 27:16	3	**t. E♭**	**151.704**		t. mi. third 32:27	294.135		t. E♭ = j. D♯ +&– I, t. D♭ = j. C♯ +&– II, t. B♭ = j. A♯ +&– IV, t. A♭ = j. G♯ +&– V, t. E VII, t. B III, t. F♯ = j. G♭ VI, t. a♭♭ = j. g VI, t. d♭♭ = j. c III, t. g♭♭ = j. f VII.
	903.911	s. j. dim. seventh		**j. D♯**	**151.875**		b. aug. j. whole tone	296.089	t. E 15 t. B 5 t. C 1215	
t. D 1/15 t. G 1/5	884.359	j. ma. sixth 5:3		**j. E♭**	**153.600**		j. mi. third 6:5	315.641		j. E♭ = t. D♯ +&– I, j. D♭ = t. C♯ +&– II, j. B♭ = t. A♯ +&– IV, j. A♭ = t. G♯ +&– V, j. E = t. D♯♯ VII, j. B = t. A♯♯ III, j. F♯ = t. E♯♯ VI, t. g VI, t. c III, t. f VII.
	882.405	t. dim. seventh 5:3.0033		**t. D♯**	**153.773**	9	t. aug. whole tone 6.0066:5	317.595		
	815.640	t. aug. fifth 8.00898:5	8	**t. F♭**	**159.820**		t. dim. fourth 5:4.00449	384.36		j. e = t. f♭ I, j. d = t. e♭♭ II, j. b = t. c♭ IV, j. a = t. b♭ V, t. F VII, t. C III, t. G VI. 16/20 = mese of the Mars mode on the fundamental c = 128 Hz.
	813.686	j. mi. sixth 8:5		**j. E**	**160.000**		j. ma. third 5:4	386.314	t. F 15 t. C 5	
t. E♭ 1/15 t. A♭ 1/5	794.134	j. aug. fifth 512:405		**j. F♭**	**161.818**		j. dim. fourth 405:256	405:866		t. E +&– I, t. D +&– II, t. B +&– IV, t. A +&– V, j. F = t. E♯ VII, j. C = t. B♯ III, j. G = t. F♯♯ VI, j. F♯ = t. G♭ VII, j. c♯ = t. d♭ III, j. g♯ = t. a♭ VI.
	792.18	t. mi. sixth 128:81		**t. E**	**162.000**	4	t. ma. third 81:64	407.820		
	790.226	j. dble dim. seventh		**j. D♯♯**	**162.182**		j. dble aug. whole tone	409:774	t. E♯ 15 t. B♯ 5	
t. D♯ 1/15 t. G♯ 1/5	770.674	s. j. mi. sixth		**h. j. E**	**164.026**		b. j. ma. third	429.326		j. E = t. D♯♯ I, j. D = t. C♯♯ II, j. B = t. A♯♯ IV, j. A = t. G♯♯ V, t. g♯ = j. a♭ VI, j. c♯ = t. d♭ III, j. g♭ = t. f♯ VII.
	768.72	t. dble dim. seventh		**t. D♯♯**	**164.210**	16	t. dble aug. whole tone	431.28		
	725.416	t. dble aug. fourth	13	**t. G♭♭**	**168.370**		t. dble dim. fifth	474.584		t. G♭ = j. F♯ VII, t. D♭ = j. C♯ III, t. A = j. G♯ VI, j. f = t. g♭♭ I, j. c♭ = t. f♭♭ II, j. c = t. d♭♭ IV, j. b♭ = t. c♭♭ V.
	723.461	j. acute fifth 243:160		**l. j. F**	**168.560**		j. grave fourth 320:243	476.539	t. G♭ 15 t. D♭ 5	
t. F♭ 1/15 t. B♭♭ 1/5	703.909	j. dble aug. fourth		**j. G♭♭**	**170.474**		j. dble dim. fifth	496.091		t. F +&– I, t. E♭ = j. D♯ +&– II, t. C +&– IV, t. B♭ = j. A♯ +&– V, j. G♭ = t. F♯ VII, j. D♭ = t. C♯ III, j. A♭ = t. G♯ VI, j. a = t. b♭ VI, j. d = t. e♭♭ III, j. g = t. a♭♭ VII. 12/16 on the generic tone c^5 = 4096 Hz.
	701.955	fifth 3:2	1	**t. F**	**170.667**		fourth 4:3	498.045		
	700.001	j. dim. sixth (s. = e.t. fifth)		**j. E♯**	**170.859**		j. aug. third (b. = e.t. fourth)	499.999	t. F♯ 15 t. C♯ 5	

t. E 1/15 t. A 1/5	680.449 678.495	j. grave fifth 40:27 t. dim. sixth		**h. j. F** **t. E♯**	**172.800** **172.995**	 11	j. acute fourth 27:20 t. aug. third	519.551 521.505		j. F = t. E♯ I, j. E = t. D♯ II, j. C = t. B♯ IV, j. B♭ = t. A♯ +&–V, j. F♯ = t. E♯♯ VII, j. C♯ = t. B♯♯ III, t. g VII, t. d III. t. a VI.
	611.731 609.777	t. aug. fourth 729:512 j. dim. fifth 64:45	6	**t. G♭** **l. j. F♯**	**179.797** **180.000**		t. dim. fifth 1024:729 j. aug. fourth 45:32	588.269 590.223	 t. G 15 t. D 5 t. C 45	t. G♭ = j. F♯ I, j. e = t. f♭ II, j. C♯ = t. D♭+&– IV, j. b = t. c♭ V, t. G VII, t. D III, t. A VI, j. c♭ = t. f♭♭ III, j. b♭ = t. c♭♭ VI.
t. F 1/15 t. B 1/5 t. C 1/45	590.223 588.269	j. aug. fourth 45:32 t. dim. fifth 1024:729		**j. g♭** **t. f♯**	**182.044** **182.250**	 6	j. dim. fifth 64:45 t. aug. fourth 729:512	609.777 611.731		j. G♭ = t. F♯ +&– I, t. E +&– II, j. D♭+&– IV, t. B +&–V, j. G = t. F♯♯ VII, j. D = t. C♯♯ III, j. A = t. G♯♯ VI, j. g♯ = t. a♭ VII, t. c♭ = j. d♯ III, t. l♭, j. a VI.
t. E♯ 1/15 t. A♯ 1/5	566.764 564.810	b. j. aug. fourth t. dble dim. sixth		**h. j. F♯** **t. E♯♯**	**184.582** **184.736**	 18	b. j. dim. fifth t. dble aug. third	633.236 635.190		j. F♯ = t. E♯♯ I, j. E = t. D♯♯ II, j. C♯ = t. B♯♯ IV, j. B = t. A♯♯ V, t. g♯ = j. a♭ VII, t. d♯ II, t. b♭ = j. a♯ VI.
	521.505 519.551	t. aug. third j. acute fourth 27:20	11	**t. A♭♭** **l. j. G**	**189.416** **189.630**		t. dim. sixth j. grave fifth 40:27	678.495 680.449	 t. A♭ 15 t. E♭5	j. g = t. a♭♭ I, j. f = t. g♭♭ II, j. d = t. e♭♭ IV, j. c = t. d♭♭ V, t. A♭ = j. G♯ VII, t. E♭ = j. D♯ III, t. B♭ = j. A♯ VI.
t. G♭ 1/15 t. C♭ 1/5	499.999 498.045 496.091	j. aug. third (b. = e.t. fourth) fourth 4:3 j. dble dim. fifth		**j. A♭♭** **t. G** **j. F♯♯**	**191.783** **192.000** **192.216**	 1	j. dim. sixth (s. = e.t. fifth) fifth 3:2 j. dble aug. fourth	700.001 701.955 703.909	 t. G♯ 15 t. D♯5	t. G +&– I, t. F +&– II, t. D +&– IV, t. C +&–V, j. A♭ = t. G♯ VII, j. E♭ = t. D♯ III, j. B♭ = t. A♯ VI, j. a = t. b♭ VII, j. e = t. f♭ III, j. b = t. c♭ VI. 16/24 = mese of the Venus mode on the fundamental c = 128 Hz.
t. F♭ 1/15 t. B 1/5	476.539 474.584	j. grave fourth 320:243 t. dble dim. fifth		**h. j. G** **t. F♯♯**	**194.400** **194.620**	 13	j. acute fifth 243:160 t. dble aug. fourth	723.461 725.416		j. G = t. F♯♯ I, j. F = t. E♯ II, j. D = t. C♯♯ IV, j. C = t. B♯ V, t. a VII, t. e III, t. b VI.
	407.820 405.866	t. ma. third 81:64 j. dim. fourth 405:256	4	**t. A♭** **j. G♯**	**202.272** **202.500**		t. mi. sixth 8:5 j. aug. fifth 512:405	792.180 794.134	 t. A 15 t. E 5 t. C 405	t. A♭ = t. G♯ +&– I t. G♭ = j. F♯ +&– II, t. E♭ = j. D♯ +&– IV, t. D♭ = j. C♯ +&–V, t. A VII, t. E III, t. B VI, j. b♭ = t. c♭♭ VII, j. f = t. g♭♭ III, j. c = t. d♭♭ VI.

(*Continued*)

Table 22 (cont.)

Undertone proportion*	Interval to c^1 = 256 Hz cents	name	Lower fifth of C	Tones name	Hz	Higher fifth of C	Interval to c = 128 Hz name	cents	Overtone proportion	Degrees of the major, minor scales and aulos modes
t. G 1/15 t. C 1/5	386.314	j. ma. third 5:4		**j. A♭**	**204.800**		t. mi. sixth 8:5	813.686		j. A♭ = t. G♯ +&– I, j. G♭ = t. F♯ +&– II, j. E♭ = t. D♯ +&– IV, j. D♭ = t. C♯ +&– V, t. c VI, t. f III, t. b♭ = j. a♯ VII. 10/16 on the generic tone c^5 = 4096.
	384.360	t. dim. fourth 5:4.00449		**t. G♯**	**205.031**	8	t. aug fifth 8.00898:5	815.64		
	317.595	t. aug. second 6.0066:5	9	**t. B♭♭**	**213.092**		t. dim seventh 5:3.0033	882.405		j. a = t. b♭♭ I, j. g = t. a♭♭ II, j. e = t. f♭ IV, j. d = t. e♭♭ V, t. B♭ VII, t. F III, t. C VI.
	315.641	j. mi. third 6:5		**j. A**	**213.333**		j. ma. sixth 5:3	884.359	t. B♭15 t. F 5	
t. A♭ 1/15 t. D♭ 1/5	296.089	j. aug. second		**j. B♭♭**	**215.757**		j. dim. seventh	903.911		t. A +&– I, t. G +&– II, t. E +&– IV, t. D +&– V, j. B♭ = t. A♯ VII, j. F = t. E♯ III, j. C = t. B♯ VI, j. b = t. c♭ VII, j. f = t. g♭♭ III, j. c♯ = t. d♭ VI.
	294.135	t. mi. third 32:27		**t. A**	**216.000**	3	t. ma. sixth 27:16	905.865		
	292.181	j. dble dim. fourth		**j. G♯♯**	**216.244**		j. dble aug. fifth	907.819	t. A♯ 15 t. E♯ 5	
t. G♯ 1/15 t. C♯1/5	272.629	s. j. mi. third		**h. j. A**	**218.699**		b. j. ma. sixth	927.371		j. A = t. G♯♯ I, j. G = t. F♯♯ II, j. E = t. D♯♯ IV, j. D = t. C♯♯ V, t. b VII, t. f♯ = j. g♭ III, t. c♯ = j. d♭ VI.
	270.675	t. dble dim. fourth		**t. G♯♯**	**218.946**	15	t. dble aug. fifth	929.325		
	231.173	v.b. j. whole tone 8:7		**v.l. j. B♭**	**224.000**		v.s. j. mi. seventh	968.827	t. C 7	t. d♭♭ = j.♭♭ I, t. g♭♭ = j. f IV, t. f♭♭ = j. e♭ V, t. D♭ = j. C♯ VI, t. G♭= j. F♯ III. 16/18 = mese of the Moon mode on the fundamental c = 128 Hz.
	227.370	t. dble aug. prime	14	**t. C♭♭**	**224.492**		t. dble dim. octave	972.630		
	225.416	b. j. whole tone		**l. j. B♭**	**224.746**		s. j. mi. seventh	974.584	t. C♭15 t. G♭ 5	
	203.910	t. whole tone 9:8	2	**t. B♭**	**227.555**		t. mi. seventh 16:9	996.09		t. B♭ = j. A♯ +&– I, t. A♭ = j. G♯ +&– II, t. F +&– IV, t. E♭ = j. D♯ +&– V, t. B VII, t. F♯ = j. G♭ III, t. C♯ = j. D♭ VI, t. d♭♭ = j. c VII, t. a♭♭ = j. g III, t. d♭♭ = j. d VI. 9/16 on the generic tone c^5 = 4096 Hz.
	201.956	j. dim. third 6:5.339		**j. A♯**	**227.812**		j. aug. sixth 5. 339:3	998.044	t. B♯ 15 t. F♯ 5 t. C 3645	
t. A 1/15 t. D 1/5	182.404	j. whole tone 10:9		**j. B♭**	**230.400**		j. mi. seventh 9:5	1017.596		t. A♯ = j. B♭ +&– I, t. A♭= j. G♯ +&– II, t. F +&– IV, t. E♭ = j. D♯ +&– V, t. B VII, t. E♯♯ = j. F III, t. B ♯♯ = j. C♯ VI, t. c VII, t. g III, t. d VI.
	180.450	t. dim. third 10:9.0102		**t. A♯**	**230.660**	10	t. aug. sixth 9.0102:5	1019.550		
	113.685	t. apotomé 2187.2048	7	**t. C♭**	**239.729**		t. dim. octave 4096:2187	1086.315		t. c b = j. b I, t. b♭ = j. a II, t. g♭ = j. f♯ IV, t. f b = j. e V, t. C VII, t. D VI, t. f♭♭ = j. e♭ VI.
	111.731	j. semitone 16:15		**j. B**	**240.000**		j. ma. seventh 15:8	1088.269	t. C 15 t. G 5	
t. B♭ 1/15 t. E♭ 1/5	92.179	j. limma 135:128		**j. C♭**	**242.726**		j. dim. octave	1107.821		t. B +&– I, t. A +&– II, t. F♯ = j. G♭ +&– IV, t. E +&– V, t. B♯ = j. C VII, t. F♯♯ = j. G III, t. C♯♯ = j. D VI, t. a♭ = j. g♯ III, t. d♭ = j. c♯ VII, t. e♭ = j. d♯ VI.
	90.225	t. limma 256:243		**t. B**	**243.000**	5	t. ma. seventh 243:128	1109.775		
	88.271	j. dble dim. third		**j. A♯♯**	**243.273**		j. dble aug. sixth	1111.729	t. B♯15 t. F♯♯ 5	

t. A♯ 1/15 t. D♯ 1/5	68.718 66.755	s. j. semitone t. dble dim. third		**h. j. B** **t. A♯♯**	**246.037** **246.315**	 17	b. j. ma. seventh t. dble aug. sixth	1131.282 1133.245		j. B = t. A♯♯ I, j. A = t. G♯♯ II, j. F♯ = t. E♯ IV, j. E = t. D♯♯ V, t. d♯ = j. e♭ VI, t. g♯ = j. a♭ III, t. c♯ = j. d♭ VII.
  t. C♭ 1/15 t. F♭ 1/5	23.460 21.506 1.954	t. comma 531441:524228 didymus comma 81:80 j. dim. second	12	**t. D♭♭** **j. C** **j. D♭♭**	**252.554** **252.840** **255.712**		t. aug. seventh 1048576:531441 s. j. octave 160:81 j. aug. seventh	1176.54 1178.494 1198.046	t. D♭ 15 t. A♭ 5	t. d♭♭ = j. c I, t. c♭ = j. b♭ II, t. a♭♭ = j. g IV, t. g♭♭ = j. e V, t. D♭ = j. C♯ VII, t. A♭ = j. G♯ III, t. e♭ = j. D♯ VI.
	0.000	prime		**t. C**	**256.000**		octave	1200.000		t. C +&– I, t. B +&– II, t. G +&– IV, t. F +&– V, t. C♯ = j. D♭ VII, j. a = t. b♭ III, j. e = t. f♭ VI. 16/16 mese of all aulos modes of the generic tone c^5 = 4096 Hz.

* e.g. j. D♭♭ = 1/15 undertone of t. C♭.

Abbreviations:

	intervals		tones	scales
	t. = true	mi. = minor	t. = true	+&– = major and minor
	j. = just	v.s. = very small	j. = just	I, II, III... = degrees
	dble = double	v.b. = very big	l. = low	fractions: modal degrees
	s. = small	aug. = augmented	h. = high	capitals = major
	b. = big	dim. = diminished	v.l. = very low	lower case = minor
	ma. = major	e.t. = equal tempered	v.h. = very high	

descending interval names and cents with c^1 = 256 Hz to the left. Next on the left are the undertone degrees of the true tone on which the just tone in question is to be found, and to the right the overtone degree. On the extreme right all degrees of the scale are given on which the tone in question sounds. The abbreviations are given at the end of the table.

Comparison of the 45 tonics of the scales in the circle of fifths as shown in Table 21 with Table 22 shows that these are now limited to the 35 true tones as we know them from antiquity, from F♭♭ to B♯♯. In order to illustrate this, the arrangement of Table 21 is repeated below in Table 23 with each true tone having the corresponding just tone added below it. This new arrangement shows not only 11 but 21 doublings-up of the true tones which have been numbered from F♭♭ (1) to B♯♯ (35). This helps us to see that the true F♯ (No. 22), which only appears twice in Table 21, is now the tonic of three scales—true F♯ major, just G♭ (= true F♯) major and true F♯ minor. On the other hand true G♭ (No. 10) is the tonic of two scales. This doubling up means that the circle of fifths contains 52 instead of 56 different scales, as the four pairs, Nos. 13, 14, 22 and 23, result in four and not eight scales.

Table 23
The 35 pairs of tonics in the circle of fifths

True major scales			Parallel just minor scales		
10	true G flat			just E flat	= true F double flat 1
	just G flat	= true F sharp 22	13	true E flat	
11	true D flat			just B flat	= true C double flat 2
	just D flat	= true C sharp 23	14	true B flat	
12	true A flat			just F	= true G double flat 3
	just A flat	= true G sharp 24	15	true F	
13	true E flat			just C	= just D double flat 4
	just E flat	= true D sharp 25	16	true C	
14	true B flat			just G	= true A double flat 5 (grave fifth)
	just B flat	= true A sharp 26	17	true G	
15	true F			just D	= true E double flat 6
	just F	= true E sharp 27	18	true D	
16	true C			just A	= true B double flat 7
	just C	= true B sharp 28	19	true A	
17	true G			just E	= true F flat 8
	just G	= true F double sharp 29	20	true E	
18	true D			just B	= true C flat 9
	just D	= true C double sharp 30	21	true B	
19	true A			just F sharp	= true G flat 10
	just A	= true G double sharp 31	22	true F sharp	
20	true E			just C sharp	= true D flat 11
	just E	= true D double sharp 32	23	true C sharp	
21	true B			just G sharp	= true A flat 12
	just B	= true A double sharp 33	24	true G sharp	
22	true F sharp			just D sharp	= true E flat 13
	just F sharp	= true E double sharp 34	25	true D sharp	
23	true C sharp			just A sharp	= true B flat 14
	just C sharp	= true B double sharp 35	26	true A sharp	

The term 'enharmonically interchangeable' is only correct for pairs of tones consisting of a just and a true tone that have the same inherent quality (see also Tables 25 and 33).

A connection between the old traditional and modern knowledge is thus found in the sphere of tones. To avoid misunderstanding, it should be said once again that only the named 5th and 15th overtones and the 1/5th and 1/15th undertones correspond with the 35 true flat, double flat, sharp and double sharp tones known from antiquity. All other degrees of these two just rows of C, e.g. 1/11, 1/13, 11 or 13, etc., show no connection to the named 35 true-tone rows.

A further fact to be seen in Table 22 is the unexpected cross-over between undertones and true sharp tones and overtones and true flat tones. This highly surprising fact, important for understanding the single qualities of the just tones and intervals, may be illustrated with two examples:

1) By means of Table 25 (see pp. 119–18), which graphically presents the connection between seven of the overtones of true C under consideration and two overtones each of true F, true B♭, true E♭ and true A♭ with the corresponding lower fifths of C, as well as those of seven undertones of true C and two undertones each of true G, true D, true A and true E with the corresponding higher fifths of C.
2) By the two just major and two just minor scales, which begin on true tones, as shown below in Table 24, giving the degrees of the scale in correspondence to the true and just tones. The true tone for which the just tone is either the 5th overtone or 1/5th undertone, correspondingly the 15th higher fifth or 1/15th lower fifth of C, is also indicated. The Hz and interval proportions are given in accordance with Table 22.

According to the above (in contrast to Tables 3 and 4), the structure of the just major and diatonic minor scales can be described as follows: both are made up of two groups of true tones from the true-tone row of C.

In the just major scales the first group consists of tonic, subdominant (1st lower fifth), dominant (1st higher fifth), and supertonic (2nd higher fifth). The second group consists of the 7th lower fifth of the tonic (leading note), its 8th lower fifth (mediant) and its 9th lower fifth (submediant).

The diatonic minor scale has the same first group. The second group consists of the 8th higher fifth of the tonic (submediant), its 9th higher fifth (mediant) and 10th higher fifth (leading note).

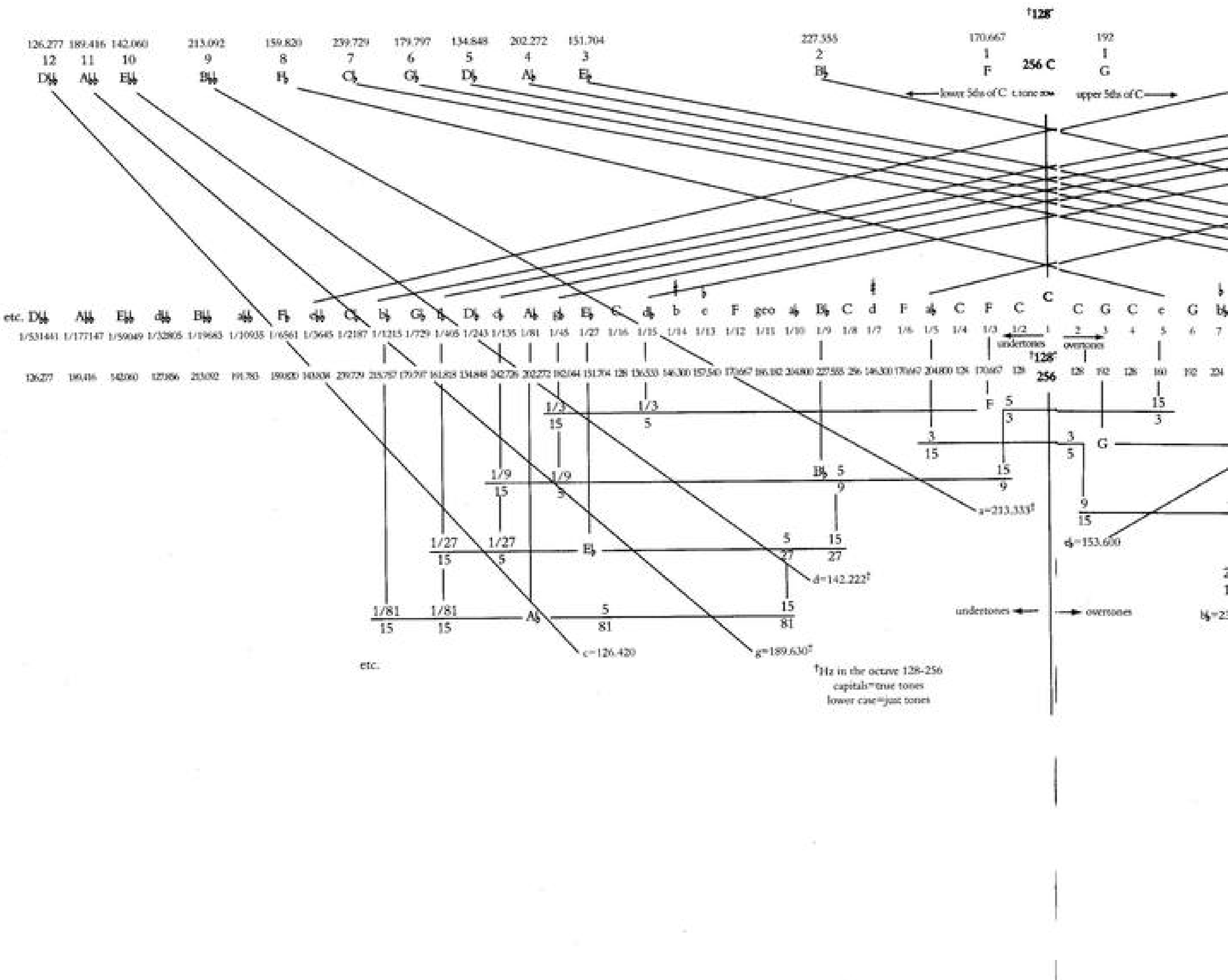
A further question arises concerning the inherent qualities of the just tones that act as tonic and scale-degree tones in the scales of the circle of fifths and make up the beat-free imperfect consonances. On

Table 24
A comparison between the degrees of the just major and minor scales beginning on true tones

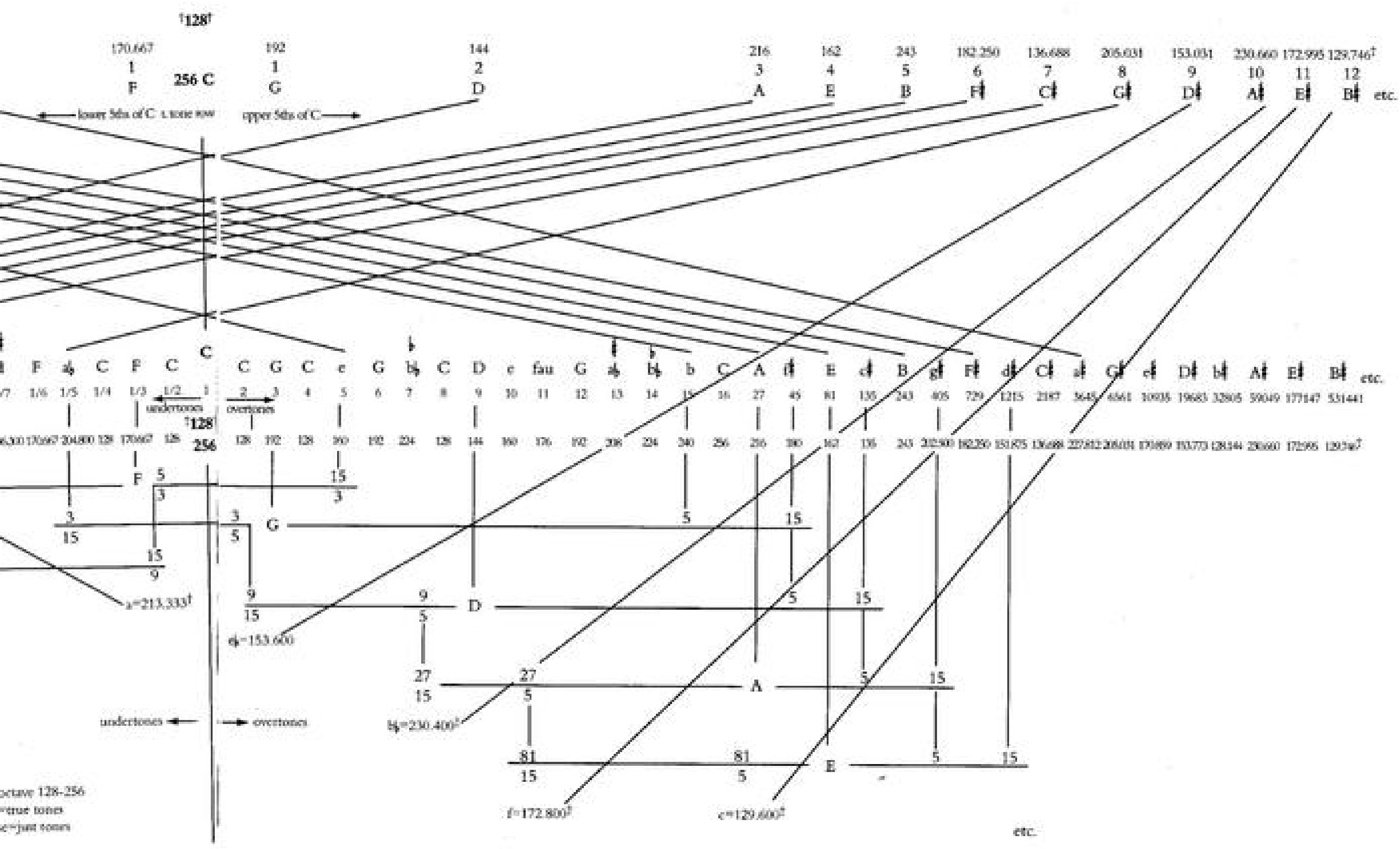
Degree:	I	II	III	IV	V	VI	VII
			true G♭ 5th ot.			true C♭ 5th ot.	true D♭ 5th ot.
Major:	true G♭	true A♭	just B♭ = true C♭♭	true C♭	true D♭	just E♭ = true F♭♭	just F = true G♭♭
			true C 14th lf.			true C 15th lf.	true C 13th lf.
			true A 5th ot.			true D 5th ot.	true E 5th ot.
	true A	true B	just C♯ = true D♭	true D	true E	just F♯ = true G♭	just G♯ = true A♭
			true C 5th lf.			true C 6th lf.	true C 4th lf.
			true G 1/5th ut.			true C 1/5th ut.	true D 1/5th ut.
Minor:	true C	true D	just Eb = true D♯	true F	true G	just A♭ = true G♯	just B♭ = true A♯
			true C 9th hf.			true C 8th hf.	true C 10th hf.
			true A♯ 1/5th ut.			true D♯ 1/5th ut.	true E♯ 1/5th ut.
	true D♯	true E♯	just F♯ = true E♯♯	true G♯	true A♯	just B = true A♯♯	just C♯ = true B♯♯
			true C 18th hf.			true C 17th hf.	true C 19th hf.

Abbreviations: ot. = overtone; lf. = lower fifth; ut. = undertone; hf. = higher fifth

Table 25
Enharmonic correspondence. Graphical presentation of the reciprocal transition from just to true tones



e tones


128
256 C
lower 5ths of C
upper 5ths of C
undertones
overtones
a=213.333
e♭=153.600
b♭=230.400
f=172.800
c=129.600
undertones
overtones
octave 128-256
=true tones
=just tones
etc.

the one hand they appear as just tones, on the other hand as true tones belonging to the parts of the true-tone row of C that is made up of sharpened or flattened (altered) planetary tones. Playing these singly or doubly altered tones on a monochord one immediately notices how much the quality of these tones is affected by the alterations. The elevated tones are aurally experienced as being over-tensed, the lowered tones as progressively more cramped. In addition to their individual qualities, the sharpened true tones thus commonly display something flaming and proud, i.e., a luciferic colouring; the flattened true tones commonly have a hardened, biting, i.e., ahrimanic colouring, in addition to their individual qualities.[26, 27, 28, 29]

The aurally observable change in the individual quality of the following two groups of tones illustrates the above very nicely and will play an important role in conceptually grasping present-day concert pitches.

			↗ true A♯♯ = 492.630 Hz
Luciferically coloured tones:		↗ true A♯ = 461.320 Hz ↗	
Unchanged tones:	true A = 432 Hz ↗	true Belis = 458.206 Hz	true B = 486 Hz
Unchanged tones:	true A = 432 Hz ↘	true Alis = 407.290 Hz	true G = 384 Hz
Ahrimanically coloured tones:		↘ true A♭ = 404.544 Hz ↘	
			↘ true A♭♭ = 378.932 Hz
			↗ true E♯♯ = 369.427 Hz
Luciferically coloured tones:		↗ true E♯ = 345.990 Hz ↗	
Unchanged tones:	true E = 3234 Hz ↗	true F = 34 .333 Hz	true Gelis = 362.04 Hz
Unchanged tones:	true E = 3234 Hz ↘	true Elis = 305.470 Hz	true D = 288 Hz
Ahrimanically coloured tones:		↘ true E♭ = 303.408 Hz ↘	
			↘ true E♭♭ = 284.120 Hz

We will now consider the intervals between just and true tones and C that are shown in Table 26. This table is an extended form of Table 2.

Part A of Table 26 shows the true scale intervals proceeding stepwise according to the degree numbers of the twelfth-tone row (see chapter 4). Starting from C, the six pairs of complementary (inverted) intervals—4th and 5th, true F and true G; major second and minor 7th, true D and true B etc.—give the commonly known tonics of the scales.

Part B of Table 26 lists the singly altered and Part C the doubly altered true tones and intervals according to Table 22. The just intervals are placed next to the true intervals so that one can see that all imperfect just intervals are almost exactly the same size as certain true intervals. The maximum difference is 2 cents. 'Behind' each imperfect just consonance, no matter how beat-free it is, is a clearly dissonant true interval. Comparison between perfect fifths and tempered fifths in Part One of this book showed that 2 cents is enough to differentiate between aurally genuine and false intervals. This explains the difference in quality between the two intervals in each pair, shown below.

Behind the ascending just major 3rd is an ascending, dissonant, true diminished 4th true C:true F♭. The sharpness that is part of the 5:4 just third, in spite of its clear beauty, can never be put down to its being beat-free, but its origin becomes understandable when one knows that an ascending true diminished 4th is 'behind' it. Or, if one considers the just minor 3rd 6:5 just E♭:true C, it has something incredibly sad but also voluptuous about it. The latter characteristic cannot be explained as due to being beat-free, but becomes clear as soon as one realizes that the seductive ascending true augmented 2nd is 'behind' it. Such is the relationship between all the other just intervals and the true intervals that are behind them. Table 26 also shows that a kind of cross-over takes place, as the true diminished, i.e., ahrimanically coloured intervals appear 'behind' the just major intervals, and luciferically coloured true augmented intervals 'behind' the just minor intervals.[26, 27, 28, 29]

This important, perhaps even epoch-making observation shows that the true intervals have more vital qualities and are perhaps more original forms than the just intervals. Otherwise they could not add

Table 26
(Extension of Table 2)
The connection between true and just intervals

Degree number of the twelve-tone row: degree number proportion	tone example	Set within one octave: a) interval b) complementary interval to the octave	cents		just-row proportions	cents
		A. True diatonic intervals				
2 to 1	$g^1 : c$	a) fifth	701.955		3:2	
	$c : {}_1F$	b) fourth	498.045		4:3	
3 to 1	$d^3 : c$	a) whole tone	203.910		9:8	
	$c : {}_3B$	b) mi. seventh	996.090		16:9	
4 to 1	$a^4 : c$	a) ma. sixth	905.865		27:16	
	$c : {}_4E\flat$	b) mi. third	294.135		32:27	
5 to 1	$e^6 : c$	a) ma. third	407.820		81:64	
	$c : {}_6A\flat$	b) mi. sixth	792.180		128:81	
6 to 1	$b^7 : c$	a) ma. seventh	1109.775		243:128	
	$c : {}_7D\flat$	b) limma	90.225		256:243	
7 to 1	$b7 : {}_1F$	a) dim. fifth	588.269		1024:729	
	${}_1F : b^7$	b) aug. fourth	611.731		729:512	
	$f\sharp^9 : c$					
	$c : {}_9G\flat$					
		B. True single altered intervals		**Just intervals**		
8 to 1	$c\sharp^{11} : c$	a) apotomé	113.685	j. semitone	16:15	111.731
	$c : {}_{11}C\flat$	b) dim. octave	1086.315	j. ma. seventh	15:8	1088.269
9 to 1	$g\sharp^{12} : c$	a) aug. fifth	815.640	j. mi. sixth	8:5	813.686
		b) dim. fourth	384.360	j. ma. third	5:4	386.314
10 to 1	$d\sharp^{14} : c$	a) aug. second	317.595	j. mi. third	6:5	315.641
	$c : {}_{14}B\flat\flat$	b) dim. seventh	882.405	j. ma. sixth	5:3	884.359
11 to 1	$a\sharp^{15} : c$	a) aug. sixth	1019.550	j. mi. seventh	9:5	1017.596
	$c : {}_{15}E\flat\flat$	b) dim. third	180.450	j. small whole tone	10:9	182.404
12 to 1	$e\sharp^{17} : c$	a) aug. third	521.505	acute fourth	27:20	519.551
	$c : {}_{17}A\flat\flat$	b) dim. sixth	678.495	ernest fifth	40:27	680.449
13 to 1	$f\sharp^9 : {}_9G\flat$	a) t. comma	23.460			
	$b\sharp^{18} : c$	b) aug. seventh	1176.540			
	$c : {}_{18}D\flat\flat$					

(*Continued*)

Table 26 (cont.)

Degree number of the twelve-tone row		Set within one octave				
degree number proportion	tone example	a) interval b) complementary interval to the octave	cents		just-row proportions	cents
		C. True double altered intervals		**Modal intervals**		
14 to 1	f♯♯^{20} : c	a) dble aug. fourth	725.416			
	c : $_{20}$G♭♭	b) dble dim. fifth	474.584			
15 to 1	c♯♯^{22} : c	a) dble aug. prime	227.370	aug. whole tone	8:7	231.174
	c : ^{22}C♭♭	b) dble dim. octave	972.630	small j. mi. seventh	7:4	968.826
16 to 1	g♯♯^{23} : C	a) dble aug. fifth	929.325			
	c : $_{23}$F♭♭	b) dble dim. fourth	270.675			
17 to 1	d♯♯^{25} : c	a) dble aug. second	431.280			
		b) dble dim. seventh	768.720			
18 to 1	a♯♯^{26} : c	a) dble aug. sixth	1133.245			
		b) dble dim. third	66.775			
19 to 1	e♯♯^{26} : c	a) dble aug. third	635.190			
		b) dble dim. sixth	564.810			
20 to 1	b♯♯^{30} : c	a) aug. comma (apotomé + comma)	137.145	m. semitone	13:12	138.573
		b) dble aug. seventh	1062.855	m. seventh	24:13	1061.427

Abbreviations: j. = just, t. = true, m. = modal, dim. = diminished, aug. = augmented, dble = double, ma. = major, mi. = minor

their qualities to the latter. The true intervals can be described as being 'single-track' whereas the just intervals, even the beat-free just consonances, show a clear 'double-track' quality and can seem ambiguous.

The true diatonic intervals of Part A are free from such colouring. This also holds true for the true imperfect consonances—the true mediant, submediant and leading note interval degrees—which, as long as they are contained within the seven planetary, i.e., non-altered true tones, clearly prove to be free from ahrimanic or luciferic colouring, and are experienced by the ear to be genuine. Their musical justification and usefulness is thus comprehensively established.

In Part C the true and just intervals are so dissonant that only the mediant 13:12 and mediant 24:13 are musically applicable. All of these intervals, including the last-named pair, have ahrimanically tense and luciferically feverish colouring that is easily perceived by the human ear.

The 35 tones in Tables 22 and 26 that are members of the true-tone row of C are in an individual interval relationship to this tone. If these true tones are also just tones, they make up two intervals to C—a just interval and a true interval that is 'behind' it. These intervals and tones give the individual scales their character. 'c = 128 Hz is always the prime' is therefore valid for all scales in the circle of fifths.

It is most interesting and informative in this context that under normal circumstances instruments such as lyres and pianos hold their tuning much better when tuned to true scales or the scale of twelve fifths. This phenomenon is the more surprising as these intervals are by no means beat-free, though the true and formed intervals are aurally genuine. In spite of this, instruments adjust better to them than to being equal tempered. The phenomenon, observed in practice, might be given some thought.

c = 128 Hz as c^5 = 4096 Hz is the prime of the aulos modes

Chapter 9, on the modes and the work of Schlesinger, showed that the aulos modes belong to the Dionysian scales. Furthermore, the degrees of each mode were found to proceed from its generic tone i.e., its tonic. In chapter 18, c = 128 was seen to be the Saturn tone which is appropriate for our time and the tone which, as the 'actual C', is connected to the Sun. It follows from this that it is musically possible, and in our time even necessary, that c^5 = 4096 Hz is the common generic tone for all seven planetary aulos modes. All tones of the modes are then degrees of the undertone row of c^5 = 4096 Hz. These tones leave the modern human mind totally free so that people can work with them freely. The words of Rudolf Steiner, quoted at the beginning of chapter 18, show that such a method of working is imperative for the further development of music.

The seven planetary modes thus contain the following eight-degree tones, of which six are true tones of C.

Modal description (see Tables 5 and 18)	Tone	Undertone of C
8/16	true C	1/8
9/16	true B♭	1/9
10/16	just A♭ = true G♯	1/10
12/16	true F	1/12
14/16	high just D = true C♯♯	1/14
15/16	just D♭ = true C♯	1/15
(16/16	true C	1/16)

The two remaining tones,

11/16	modal Geo	1/11
13/16	modal E	1/13

while part of the undertone row, are not part of the true-tone row of C. But they are an integral part of the modes and increase the number of tones which are available for building our scales from 35 to 37. All degrees of the modes are part of the undertone row of C. As the aulos modes do not contain any foreign tones they can be described as just scales and show that c = 128 Hz is always the prime.

c = 128 Hz is the prime of the scale of twelve fifths

The 37 tones mentioned above are not sufficient to make a twelve-degree chromatic scale because the altered tones divide the major seconds of the diatonic true-tone scale of C into unequal minor seconds; also, if one sharpened tone and one flattened tone were placed between every two non-altered tones, the resulting scale would have 17 instead of the wished for twelve degrees. Five extra tones are needed to expand the untinged seven-degree diatonic true-tone scale into an equally untinged twelve-degree chromatic scale. The five tones need to be free from luciferic or ahrimanic colouring and to divide each of the five major seconds into two minor seconds of equal size that make aurally genuine intervals among themselves and with C. True C major, with its untinged tones and 'single-track' intervals is the only possible scale as the basis for an aurally genuine-sounding twelve-degree chromatic scale, and never a just major scale made up of tinged tones and 'double-track' intervals. That is the logical conclusion of what has been discussed so far.

The five extra tones needed prove to be the geometric mean tones discussed in chapter 13. These, together with the seven non-altered planetary tones make up the scale of twelve fifths. Its form principle was discussed in chapter 13 where it was shown how the true Dorian octachord and its ascending inversion the true C major scale were organically expanded to the scale of twelve fifths by adding the geometric mean tones. Every single degree of the scale of twelve fifths has a clear and direct 'single-track' tone and interval relationship to C as prime.

The numerical relationship of the five geometric mean tones to C still remains to be established. To do this we will go back to Table 12 and calculate it from the octaves mentioned by Plato, as follows.

Tone	**Octave**	**Geometric mean**		**Relative to c = 1 Hz**	
Gelis	C:c	$\sqrt{1\times2}$	= 1.4142		$\sqrt{2}$ = 1.4142 Hz
Delis	G:g	$\sqrt{3\times6}$	= 4.2426	(3/4)	$\sqrt{2}$ = 1.0607 Hz
Alis	D:d	$\sqrt{9\times18}$	= 12.7279	(9/8)	$\sqrt{2}$ = 1.5910 Hz
Elis	A:a	$\sqrt{27\times54}$	= 38.1838	(27/32)	$\sqrt{2}$ = 1.1932 Hz
Belis	E:e	$\sqrt{40.5\times81}$	= 57.2756	(81/64	$\sqrt{2}$ = 1.7899 Hz

Relative to c = 128		**Intervals according to Table 13 and their inversions**
Gelis	181.019 Hz	formed equal tritone
Delis	135.765 Hz	formed minor second/formed major seventh
Alis	203.645 Hz	formed minor sixth/formed major third
Elis	152.735 Hz	formed minor third/formed major sixth
Belis	229.103 Hz	formed minor seventh/formed major second

The relationship of the five true geometric mean tones and the 'single-track' nature of their intervals with C is evident from the above calculations. However, the final establishment of their qualities can only be gained with an impartially listening human ear. This can be done using the monochord described in Appendix 1. Let us play the tones true g♭ = 179.797, true f♯ = 182.250 Hz and gelis = 181.019 Hz on the monochord, for example. The true G♭ has something crystalline, hard about it; the true F♯ something flaming, seductive; true Gelis sounds as if it comes from a far distance, moving steadily. These opposites become more distinct if we compare the intervals which these make with C: true G♭ makes a true diminished fourth which one feels to be cramped; true F♯ a true augmented fourth which tears the listener out of himself; true Gelis is the equal-tempered tritone which, despite being dissonant has nothing compelling about it, it does not need to be resolved and one can easily leave it resonating. So it can be said: true Gelis holds the balance midway between the ahrimanic, pressing together true G♭ and the luciferic tearing-away true F♯. The tone itself and the interval it builds to C is therefore untouched by ahrimanic and luciferic influences. The other four true geometric mean tones and their intervals relate similarly to C and their surrounding ♯ and ♭ tones. These aural observations prove that the five true geometric mean tones are as equally untinged as the seven non-altered planetary true tones. The scale of twelve fifths offers an alternative to the interval falsification of equal-tempered tuning in the practice of music, whilst retaining all the advantages of full chromaticism.

With the greatest emphasis it must be repeated that the tones of this scale are found by experience to have their untinged, balanced quality only when tuned to the concert pitch c = 128 Hz. If another pitch is used, all degrees of the scale are pulled into the inherent sphere of that pitch, as we have seen, and then are either luciferically or ahrimanically tinged. (See the experiences with tuning the scale of twelve fifths described at the beginning of chapter 14.)

The 35 true row tones, the two mediant tones and now the five true geometric mean tones give 42 tones in total. All the scales presented above can be produced with these 42 tones. Each tone is the bearer of a singular, inherent quality and makes an aurally genuine interval with the common prime c = 128 Hz. This again confirms Rudolf Steiner's comment that 'C is always prime.'

Gaining insight into the inherent qualities of other chamber pitches

Until now the inherent qualities of pitches such as 440 and 448 Hz have been described according to aural observations. Insight into these inherent qualities can be sought with the question: Which true tone in the true-tone row of C is 440 or 448 Hz? The latter has already been seen to be the 7th overtone of c = 128 Hz which is the pre-Christian mese of the Greek aulos Moon mode and makes the very small just seventh 7:4 interval with c. It is also the 14th lower 5th of this c, the true C𝄫. True C𝄫 is the common prime, C, flattened twice by the amount of an apotomé (113.685 cents) and therefore belongs to the sphere of strongly ahrimanically tinged tones.[26, 27, 28, 29] This would explain the very tiring effect tuning to 448 Hz has on musicians.

Conceptual confirmation of the aurally experienced inherent quality of 440 Hz is more difficult to find as this tone is not to be found amongst the 45 named tones that have a direct connection to c = 128 Hz. Another approach must therefore be found. On reflection, 440 is found to be the 4th higher octave of 55 Hz, which in turn is the 55th overtone of $_6C$ = 1 Hz. The 440:256 interval is an overly large 6th of 937.632 cents which is so dissonant that it is practically useless for music; 440 therefore does not belong to the tone world of c = 128 Hz. Also, 55 Hz is the 5th overtone of fau = 11 Hz (a 5:4 just major third interval), which was described as being a solemn rolling boil in chapter 18; in other words, it has to do with desires, wishes and passions, i.e., Lucifer. These insights help to make the characteristic quality of 440 Hz understandable; the connection with fau means great superficial beauty, connected with a powerful hidden tendency to incite.[26, 27, 28, 29]

These two examples show how aurally experienced singular tone qualities can be conceptually understood by examining their relationship to c = 128 Hz and other tones. This in turn further proves C to be always the prime. It follows that the concert pitches 440 and 448 Hz can influence human development in an extremely negative way. It would be progressive to avoid concert pitches of this kind.

Part Four

FURTHER ASPECTS

22 The Secret of the Octave, Open Fourths and Fifths, and Resonating Difference Tones

In the years since the first edition of this book appeared it has been possible to develop a second new method of tuning the scale of twelve fifths that gives a more beautiful, rich, and an even truer sound than the original method. The new method uses minimally enlarged octaves as well as minimally enlarged beat-free fourths and fifths where the sound of the difference tones is not only silvery but strong and sonorous. It will be presented in detail in chapter 24. Only a few pertinent points will be considered here.

Any string player knows the following phenomenon. When an open string is played together with its double octave, the upper tone is intoned so as to sound correct, i.e., so that the ear can agree with it. If after several repetitions the upper tone is played as a harmonic on the same string as the fundamental, the musician finds to his great surprise that this tone sounds too low in a way that is truly pitiful. No matter how often the experiment is repeated, the result is always the same. The harmonic of the fundamental's double octave sounds too low compared to the fundamental. It is of course possible for the musician to stop the tone at the same pitch as the harmonic, but the ear experiences both as too low compared to the fundamental. The experiment shows that the ear experiences a genuine double octave to be somewhat larger than the just double octave of the harmonic.

The phenomenon is initially perplexing. Whether or not it is objective or subjective will be left open for the moment. However, musicians have certainly experienced this countless times and experience has shown that they all play the double octave fairly exactly the same size. This alone may suggest the phenomenon to have an objective basis.

The same may be observed with the single octave. Here the difference in size between the stopped octave and the harmonic is no longer so great, though it is certainly still noticeable. A violinist, for example, would never play the octaves at the beginning of Beethoven's *Violin Concerto* beat-free (i.e., exactly 2:1), but always somewhat larger, in keeping with the aural experience.

Another practical outcome of the phenomenon which has already been investigated is the 'stretching' of the octave when tuning pianos. This is usually said to be necessary in textbooks because of the specific construction and characteristics of the instrument or because of the disharmony of the strings.[74] This does not adequately explain the sometimes considerable stretching of octaves. It does, however, confirm a verbal communication by Rudolf Steiner to Schlesinger that the true octave is minimally larger than 2:1. (See Tables 27 and 33.)

Further observations can be made with fourths and fifths on resonant instruments. Grand pianos, harpsichords and pipe organs are best for first attempts because of the required precision. Let us begin with the beat-free perfect fifth. It sounds peaceful; almost solid. The lower octave of the fundamental hums along, totally absorbed in the sound, giving a silvery colouring to the whole. If one minimally enlarges the fifth—by either raising the pitch of the higher, or lowering that of the bottom tone—the whole texture is set in motion. First of all the interval vibrates violently, then it opens up and the fifth sounds peaceful and clear once again but also wide. The difference tone emerges sonorously and sounds stable. Some people recognize the openness of the interval by a relaxation in the area of the diaphragm. If the fifth is enlarged further, the difference tone begins to beat, the sound loses its coherence and the beats dominate. If one makes the fifth smaller than perfect, it immediately loses its lustre and strength and grows milder and more inward. When the interval is smaller than a perfect fifth, the difference tone does not stabilize, being weak at first then suddenly breaking off. Where the beating sets in, the interval is roughly the same size as the equal-tempered fifth. If one makes it still smaller, the fifth will be utterly calm again (see third series in chapter 23).

Much the same applies for the fourth, though a reduction in its size hardly comes into consideration. Theoretically the difference tone of the fourth sounds a fifteenth below the higher tone, but in reality the lower octave is often heard as well. As a rule, the fourth first opens after the beat-free area. It is especially with open fourths that the difference tone adds strength and width to the sound right into the lower base regions. If it is enlarged just a little bit more, the difference tone begins to beat and the interval acquires an initial sharpness.

Following up the references to and investigations of difference tones, one finds that the first description of difference tones was given in the eighteenth century by the violinist Tartini[75] and the organist Sorge.[76] About a hundred years later Helmholtz investigated the phenomenon further. He differentiated between difference tones that are objective, i.e., present in the air, and subjective, i.e., created by the human ear. Despite all the care Helmholtz took in investigating the beats and difference tones, his results are not sufficient to penetrate the complicated conditions and fine details of the phenomena described above.

Organ builders have the greatest experience handling difference tones in practice. As the following quotations will show, the organ builders have made use of difference tones for more than a century not only to enrich the sound of large pipes (32 and 16 foot) but especially in using an 'acoustic bass' as a substitute for longer pipes where finances are low or there is lack of space; this involves pairs of smaller organ pipes tuned to each other so that their difference tone sounds at the same pitch as the tone of the larger pipe. In his large two-volume work on organ building, Audsley wrote at the beginning of the twentieth century:[77]

> The most striking proof of this latter fact that can be given, in connection with the tonal structure of the organ, is the creation of the so-called 'acoustic bass', of 32 ft pitch, by the simultaneous sounding of two pipes of 16 ft and $10\frac{2}{3}$ ft speaking length respectively. It will be found that the differential tone is an octave lower in pitch than the deeper of the two generating tones, thus:

Generating tones	Differential tone
CCC 16 ft – GGG $10\frac{2}{3}$ ft	CCCC 32 ft tone
32 Hz 48 Hz	16 Hz

> Considerable use has been made of this differential tone in the appointment of the pedal department of organs; and although the deep bass tone so produced cannot be compared to that yielded by an independent stop of 32 ft pitch, it is of some value in its power of enriching and adding gravity to the tonality of the pedal organ . . .
>
> Carrying our investigations just one step farther, we come to the differential tone produced by the simultaneous sounding of the ranks which represent the second and third upper partials of the unison prime, namely, the twelfth and super-octave or fifteenth. These ranks of pipes stand at the interval of a perfect fourth apart; and their differential tone is precisely the same as that produced by both the pairs of ranks previously spoken of, and which stand, respectively, at the intervals of a perfect octave and a perfect fifth apart. Accordingly we find the differential tone of the twelfth and fifteenth is in unison with the prime tone thus:

Prime tone	Generating tones	Differential tone
CC 8 ft	G $2\frac{2}{3}$ ft – c^1 2 ft	CC 8 ft tone
64 Hz	192 Hz 256 Hz	64 Hz

In the standard work for organ builders written in the German language, Ellerhost writes about 'combination tones':[78]

> Further tones arise under certain circumstances when two tones sound at the same time. They are called combination tones because they arise from the combination of single tones. If one plays the

tones p and q, then tones p − q and 2q − p come up very strongly; considerably less audible are 2p − q, 3q − 2p, 2p − 2q, p + q. The new tones are therefore difference and summation tones. Combination tones are partly objective (as vibrations in the air) and partly subjective (in the ear of the observer), and arise through superposition of simple primary tones.
The frequency of the difference tones is therefore the exact difference between the frequency of the primary tones. Difference tones are easier to hear when the primary tones are weak in overtone (stopped).* One first plays the lower of the two primary tones, e.g. c^1 = 258.7 Hz, and then the higher, e.g. the perfect fifth g^1 = 388 Hz [note: perfect and not equal-tempered!]. One then hears the difference tone 388 − 58.7 = 129.3 Hz, i.e., 'c', sounding faintly. The first overtones, if well developed, also produce difference tones. The development of such difference tones means that the organ builder needs to be very careful in arranging, measuring and intoning the aliquots, the fifths 5⅓ ft and 2⅔ ft and the thirds 3⅕ ft and 1⅗ ft, as well as the sevenths and ninths. The tones C (8 ft) + G (5⅓ ft) produce 97.05 − 64.7 = 32.35 Hz = $_1$C (16 ft); the fifth 5⅓ ft by natural necessity greatly thickens the sound unless kept very faint. The most important difference tone comes when 1) a tone and its perfect fifth sound together; a 10⅔ ft + 16 ft tone give the 'acoustic' 32 ft tone . . .

Both texts clearly show that effectively employed in organ building, difference tones are as objective as all other tones. Our observations are in agreement with this. They show that the 'perfect' intervals—fourths, fifths, octaves and fifteenths—have different sizes where they sound beat-free and that their difference tones sound different according to the different interval sizes. The exact sizes are still unclear because the deviation from the perfect proportions 3:2, 4:3, 2:1 and 4:1 are only mentioned in the literature when they are large.

* 'Stopped' or 'covered' here means a soft-toned metal 'stopped' organ stop of small scale. It comes fairly close to a flute quality of sound.

23 Measuring Intervals and Comparison with Audio Tape

In the previous chapter three new intervals used in the new method of tuning the scale of twelve fifths were described: fourths and fifths that have an open sound, are a hair larger than perfect, with difference tones that resound strongly and sonorously (called 'open' below), and also have minimally enlarged (open) octaves. It has been attempted to measure these intervals as exactly as is physically possible. They were produced on musical instruments (piano, violin, viola, cello, organ pipes) and as sine tones on frequency generators in a physics laboratory. The attempt to use sine tones was made because it is thought that such tones could be measured more easily and exactly and therefore give clearer results than would be possible with the more complex sound of acoustic instruments.

It needed the help of many friends to carry out these experiments. The author regards it as particularly fortuitous that the physicist Dr Rudolf Cantz, who had been working in the physics laboratory of the Science Section at the Goetheanum for many years, took an interest in the experiments, placed his instruments at our disposal and also operated them. Physicist and organist Michael Jacobi also showed great interest in the experiments. Others who joined in the effort were organ builder and piano tuner Elisabeth Ahlers, violist Christian Ginat and cellists Gotthard Killian and Marcus Gerhardts.

The physics instruments and their arrangement are shown in Figs 2 and 3. Tape recordings were made using a high quality stereo microphone.[79] Parallel to the laboratory experiments cent measurements using a Langbein SQ 22 tuning set were made on the same intervals produced on musical instruments. The aim of this series of experiments was to measure the exact size of the open fourths and fifths.

First series

More than 40 individual experiments were done in an attempt to produce open fourths and fifths as sine intervals with and without using distortion on the instruments (see arrangement in Fig. 2). The frequency generator for sine intensity (3) was tuned exactly to c^1 = 256 Hz, or its octaves 512 Hz or 1024 Hz with the help of the frequency counter (2) and the oscilloscope image (5); it was checked again before each further test. The second tone of the interval—ascending fourth, f^1, f^2, f^3, and fifth, g^1, g^2, g^3, and descending fifth, f—were produced with the frequency generator (1). The perfect intervals (3:2 or 4:3) were first found and observed and then minimally enlarged. The frequency counter (2), initially the only one available, could not directly meet the requirements, so that only the perfect intervals could be determined with it and the oscilloscope image.

The important point in this first series of experiments was that beat-free open fourths and fifths with sonorous, strong differentials could not be produced with the available equipment. When the intervals were set as perfect, so that the oscilloscope image would be stable, the differential could be heard as a constant drone. If the interval was minimally enlarged, the difference tone droned but beat strongly with, and in the same rhythm as, the oscilloscope picture.

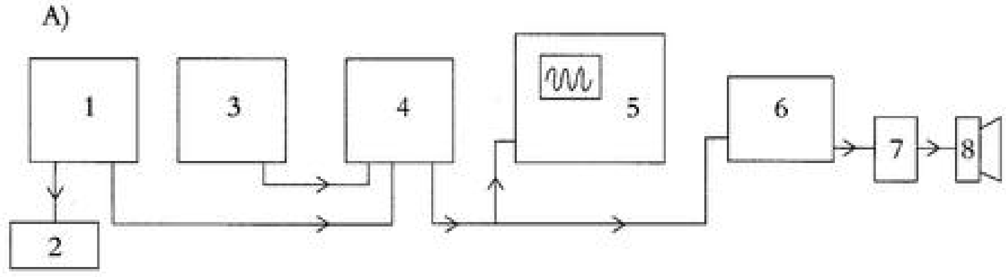
It was not possible to get the minimally enlarged intervals to open up in sound. Interesting was the fact that the difference tone of these minimally enlarged intervals clearly was too low while it was too high when the same intervals were produced on acoustic instruments.

Second series

This involved about 25 experiments. Fig. 3 shows the arrangement of the instruments (initially without the cello and micro-second counter). The two tones that make up the interval in question were produced by blowing two organ pipes. The experiment again started with perfect fourths and fifths, and the interval was then minimally enlarged by changing the pitch of one of the pipes. With the perfect intervals the differential sounded clear and silvery. As soon as enlargement reached a certain point the sound of the interval opened up and, in spite of noticeable background beating, sounded totally calm; the differential

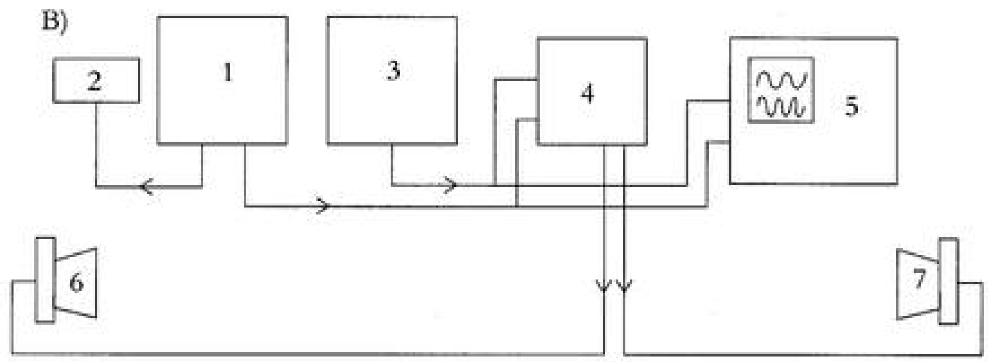
Fig. 2
Arrangement of the instruments for the first series of experiments

A)



1 Frequency generator (left, square-wave voltage, right sine-wave voltage)
2 Frequency counter
3 Frequency generator for sine voltage
4 Adding circuit, with the optional addition of a distortion diode
5 Oscilloscope
6 Power amplifier
7 Distortion diode settable from 0
8 High-fidelity loudspeaker

B)



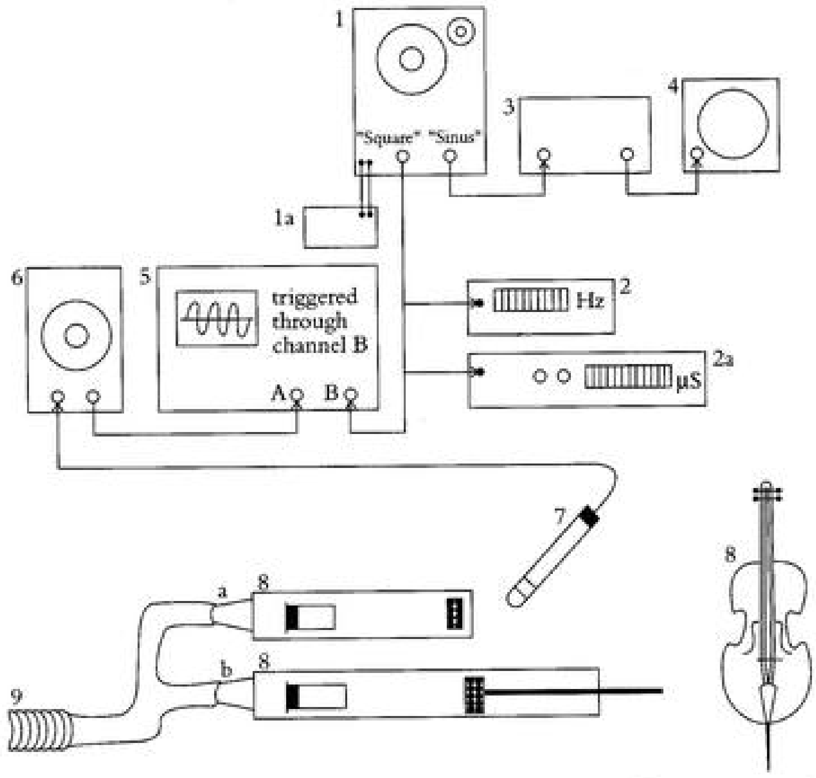
1 Frequency generator (left, square-wave voltage, right sinus-wave voltage)
2 Frequency counter
3 Frequency generator for sinus voltage, distorted by a diode lying parallel for the exit
4 Two-channel power amplifier
5 Oscilloscope
6 High-fidelity loudspeaker
7 High-fidelity loudspeaker

was strong and sonorous. As had been the case in the first series, only the inadequate frequency counter (2) was available and the size of the open fourths and fifths could not be measured.

It was nevertheless possible to make an important scientific observation. The arrangement of the instruments made it possible to produce both interval tones either on two organ pipes or one on an organ pipe and one as a sine tone on the frequency generator. When two organ pipes were used the result was as above. When the interval was produced by an organ pipe tone and a sine tone, the sound of the minimally enlarged interval did not open up, as had also happened in the experiments of the first series, and the differential only buzzed as a strongly beating noise.

Following this last observation, Dr Cantz asked if the beat-free sounding of the minimally enlarged open intervals, and the strongly sounding sonorous differential could only be produced by originally

Fig. 3
Arrangement for series 2 and 4



1	Frequency generator (left, square-wave voltage, right sine-wave voltage)	6	Low frequency amplifier
1a	Frequency fine-tuning device	7	Microphone
2	Rough frequency counter (Hz)	8	Organ pipes/cello
2a	Micro-second frequency counter	9	Air supply from bellows
3	Low frequency amplifier		
4	High-fidelity loudspeaker		
5	Oscilloscope		

The sound/tone to be measured was recorded by the microphone (7), amplified by the low frequency amplifier (6) and shown graphically on the oscilloscope (5).
The square-wave signal of the frequency generator (1) was matched to the frequency of the tone to be measured and used to trigger the oscilloscope. The sine-wave signal was only amplified and made audible to control the pitch.
The frequency of the square-wave signal from (1), which triggered the oscilloscope, was tuned to the more complicated signal from the microphone (7) by adjusting the frequency generator (1) until its picture on the oscilloscope (5) came to rest. Then the frequency, or the frequency length, could be read from the two frequency counters (2) and (2a).

playing on acoustic instruments or if the phenomenon would also be audible when a tape recording of the sounds was played. An answer to this question was sought in the following series.

Third Series

In this series of experiments, metal viola and cello strings were tuned by ear to open fifths, perfect fifths and smaller than perfect fifths and recorded on tape using the above-mentioned stereo microphone.[79]

They were replayed using a quadratic synthesis arrangement of two front and two side speakers.[80] This gave direct aural comparison of tones and intervals played directly on an acoustic instrument and electrically reproduced tones and intervals from a tape recording. The nine individual experiments were partly done at $a^1 = 443$ Hz and partly at $a^1 = 432$ Hz. The most informative was the middle one, the fifth. Marcus Gerhardts tuned the metal A string of his cello to a tuning fork pitched at 216 Hz. He then tuned the D string first to the open fifth. Continually rebowing the strings at a good forte, he raised the pitch of the D string first to a perfect fifth and then to one that was somewhat smaller and finally to an even smaller fifth that sounded utterly calm (called small fifth below). Marcus Gerhardts had a nice, even bow stroke so that the tone sounded persistently clear and the individual quality of each tone could be directly experienced. The particular difference tone could also be observed very exactly. With the open fifth it was strong and sonorous, with the perfect fifth stable and silvery, with the transitory fifths hardly audible and with the small fifth it had completely disappeared.

It was extremely interesting to compare the acoustic experiments with the tape recording. Firstly all difference tones of the different fifths were as clearly audible in the reproduction as on the acoustic instrument—a further proof of the objective nature of the difference tones in question.

A second observation concerned the inherent quality of music as it was played and of the tape recording of it played over a loudspeaker. Because of the high quality of the recording and replay equipment no outwardly aural difference was noticeable between original and 'reproduction'. It became more and more apparent that direct, live playing gave the listener a direct manifestation and expression of the essential nature of the tones played, whereas with the reproduction the essence of the tones that had been perceived in their inherent quality seemed to have been extinguished, with a grinning vacuum taking its place that was experienced as a lack of something. On the surface of this gaping void—no doubt due to the outstanding quality of the machines—the essential nature could be perceived like a shadow cast from a far distance. It was only a shadow, not the essence itself. The difference in quality between live and recorded tones we referred to earlier in this book was again confirmed.

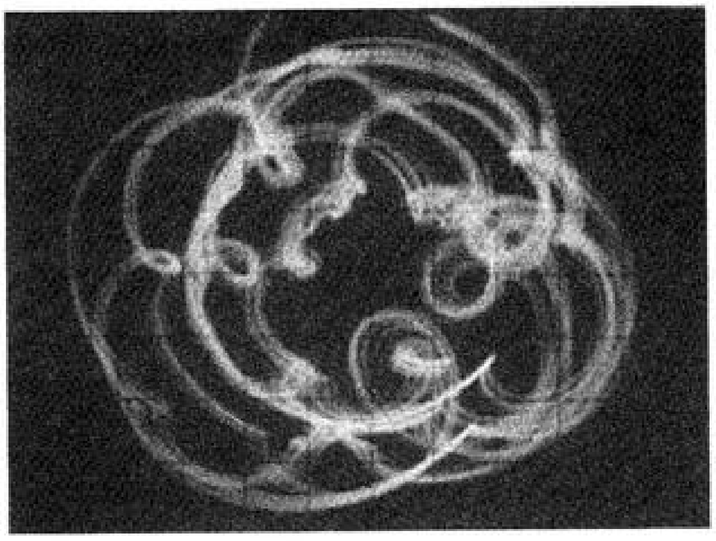
A third observation concerned the interval size of the fifth. Playing the differently sized fifths consecutively on the steel cello strings it became unmistakably obvious that not only the perfect (3:2) fifth but also the open fifth and the small fifth sounded totally calm, i.e., beat-free. In the transitions between these fifths, which differed in size, the differential beat very strongly and sounded extremely unbearable. There was not just one beat-free fifth but three! The small fifth is often used by concert musicians to tune string instruments when they use the impure intervals of an equal-tempered piano.

In a further experiment the tape recording of the fifth experiment in the third series was connected with a polar coordinated oscilloscope. The three beat-free fifths showed for a moment on the screen as beautiful, long curves, whereas with the transitory fifths a knot of chaotic, whirring lines appeared—see photographs in Fig. 4. The sequence of images was so surprising that the observers could hardly believe them. But when the images continued to be the same after many repetitions of the experiment, it had to be believed that they were a true reflection of the taped intervals. The images therefore confirmed our aural perception that two further beat-free fifths existed beside the perfect fifth. It should be pointed out, however, that the unbearable transitory fifths can most probably only be experienced with such clarity on string instruments. When tuning pianos, harpsichords etc., they can lie so close together that the tuner needs to take the greatest care to avoid them.

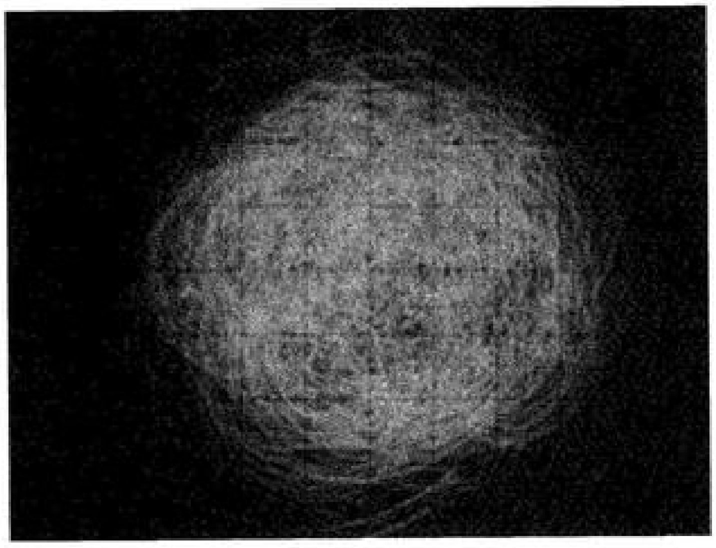
Fourth Series

After all these experiments and observations, which took over a year, it became even more important to measure the size of the open and small fifths and also the above-mentioned open fourths and the stopped (open) and harmonic (perfect) octaves. In the interim we were lucky enough to receive two further, more suitable systems for measuring intervals which finally made this possible: a micro-second frequency counter (Fig. 3, 2a) and a portable, battery-operated Langbein SQ 22 tuning set. The intervals produced

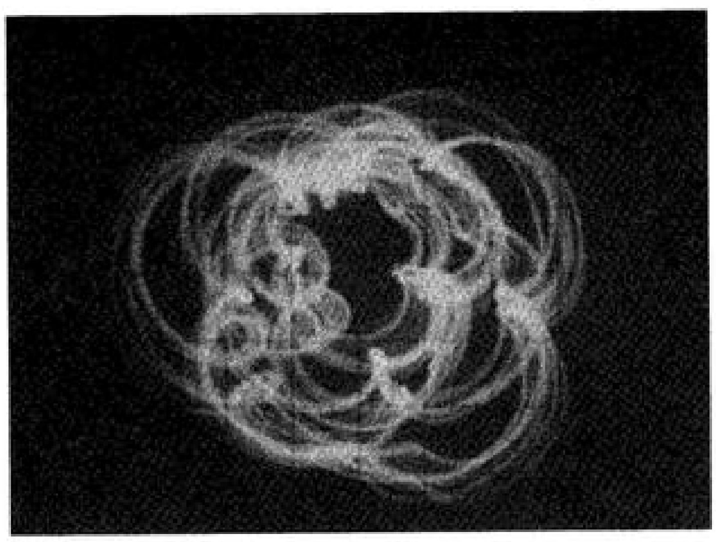
Fig. 4
Oscilloscope pictures of differently sized fifths



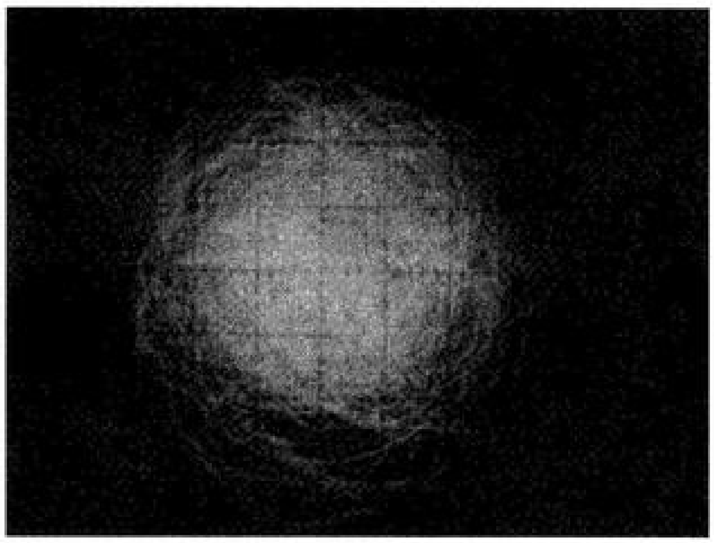
Open fifth



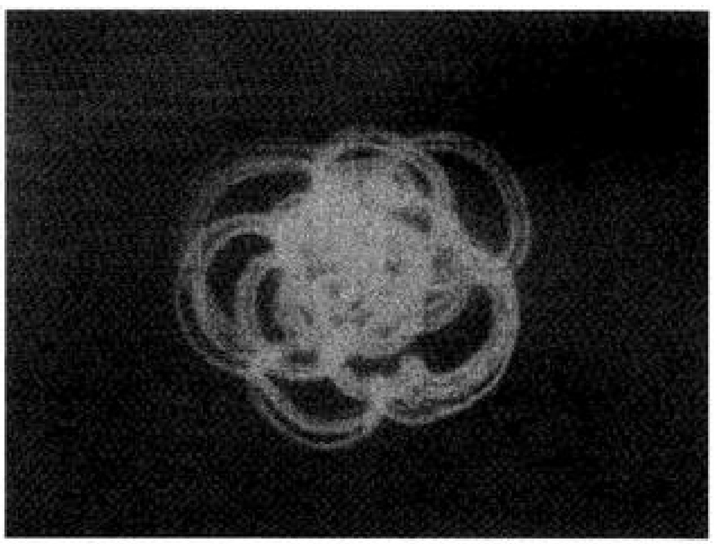
Transition



Perfect fifth



Transition



Small fifth

on organ pipes in the laboratory could be measured with the micro-second frequency counter; those played in concert halls and private homes on stringed instruments could be measured with the Langbein SQ 22 tuning set. The intervals played on violas and cellos with metal strings were measured with both appliances and used as controls. However, as it was not possible to measure the interval as such, only one of the two interval tones could be determined at a time. Each interval was therefore tuned by ear and the tones were then measured individually, with the interval size calculated from the two results.

Both systems gave remarkably similar results. The nearly one hundred measurements have been collated in Table 27. It is most interesting that the measurements of the three types of fifths have a very great difference in tolerance. While the perfect fifth has a tolerance of 1.4 cents and the small fifth of 2 cents, the open fifth sounds completely calm, with a strong and sonorous difference tone and with a difference in size of up to 3.6 cents. All of this must be carefully taken into account for correct tuning of the scale of twelve fifths.

Evaluation of the experiments

1) The tape recordings seemed to prove that difference tones are an objective acoustic and not a merely psychological phenomenon.

2) The inherent quality of the individual tones could be clearly perceived and experienced with intervals played live on stringed instruments, while on the recordings it had been extinguished and replaced by a blanket quality of a grinning vacuum. Thanks to the high quality of the recording and replay equipment, a shadow of the tone qualities could be sensed, though not the qualities themselves.
3) It was not possible to achieve open intervals using sine tones on the equipment available.
4) The sizes of the open fourths and fifths and octaves could be determined. The clear result was that all open-sounding intervals are numerically larger than the perfect intervals; the span and width over which the open intervals were measured was appreciable. When one considers that the even hundreds in the cent numbers are an exact representation of the equal-tempered intervals, it is evident from the interval measurements that an outward focus on beats or cents can at least give a first approximation, on which one will have to base oneself when tuning so long as one does not know the specific qualities of the open intervals as such. Further consequences of these determinations will be found in the tuning instructions given below.
5) Determination of the stopped and harmonic double octaves played on the cello showed that the stopped octave, which sounds correct to the ear, is a trace more than 4:1, while the measured 4:1 harmonic is clearly too low. This observation confirms Rudolf Steiner's verbal indication to Schlesinger that the true octave is a trace more than 2:1.

Table 27
Interval sizes in cents calculated from the measured frequencies of the two surrounding tones

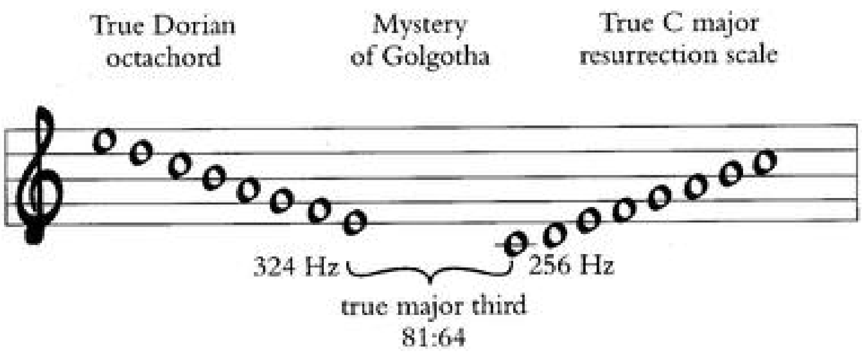
Instrument	Cents				Comments
	Open fifths, sonorous difference tone				
Grand piano	703.5	702.9	704.2		Bechstein EN
Upright piano	703.4	702.7	704.2		Sabel
Violin	703.5	703.7	704.2		open strings, steel
Viola	703.6	704	704.8		open strings, steel
Cello	702.9	703.8	706.5		open strings, steel
Organ pipes	703	704.3	705.5		stopped
	Perfect fifths, silvery difference tone				
Cello	701	702.2			open strings, steel
Organ pipes	702				stopped
	Small fifth, no difference tone				
Cello	698.5	699	700.4		open strings, steel
Note: All three fifths presented up till now are totally beat-free.					
	Open fourths, sonorous difference tone				
Grand piano	499.7	499.9	500.8	502.2	Bechstein EN
Piano	499.6	499.9	501.3	502.3	Sabel
Chord zither	499.5	499.7			
Organ pipes	499.7	499.9	500.9	502.5	stopped
Note: The fourths were only determined on organ pipes and instruments of fixed tuning.					
	Double octaves				
Grand piano	2404.6	2406.8	2411.4	2418.6	Bechstein EN
Cello, harmonic 4:1 of C	2398.7	2399.2	2399.8		c^1 sounded too low
stopped	2401.7	2404.4	2406.7		c^1 sounded right

Note: If one multiplies the widest stopped double octave on the cello by the factor 3.5 (= seven octaves), the result is the same interval as that of twelve perfect fifths; the Pythagorean comma does not apply: 2406.7 × 3.5 = 8423.45; 701.955 × 12 = 8423.46

24 A New Method of Tuning the Scale of Twelve Fifths*

The second method of tuning the scale of twelve fifths, developed since 1985, gives a further enrichment of the sound and is made up of three fundamental parts:

1) gelis^1 = 362.04 Hz† and Rudolf Steiner's pitch indications c = 128 Hz and a^1 = 432 Hz (for these pitches it is necessary to have three tuning forks that must be calibrated to within ± 0.5 Hz);
2) the open (minimally enlarged) octave, fifth and fourth intervals which, despite background beating, sound totally calm and open and have sonorous difference tones;
3) tuning in contrary direction of the Apollonian Sun scale: the pre-Christian descending direction of the true Dorian octachord and the Christian ascending direction of the true-tone C major resurrection scale (see chapter 19, Table 11 and Tables 17–19). The first and last tones of these two Apollonian scales are connected to one another by a true major third true c:true e



Thus the true major third relates directly to the Mystery of Golgotha.

The inharmonicity of each instrument is different. The results of the interval measurements as presented in Table 27 therefore showed a surprising variation in the size of the named intervals. This means that the cents can only be a rough guide and that very exact hearing and the ability to recognize these intervals is vital for successful tuning of the scale of twelve fifths. Every tone must always be fine-tuned by ear. We will therefore begin by showing how this can be done.

First tune two strings of a violin, zither, lyre etc. to a perfect fifth. Convince yourself that it is perfect by repeatedly playing the two tones and checking that the interval is calm and does not beat. The same can be done with fourths, but the tension of the string makes it inadvisable to attempt this on an instrument belonging to the violin family. If the interval in question is perfect it sounds stable and the difference tone, though not strong, is clearly audible and sounds silver.

Now increase the interval by tuning the upper string higher or the lower string lower—never change both strings at once! The interval will begin to beat intolerably. This has often shocked piano tuners so much that they hardly dared to continue. Do not give in to 'fear of beats', but calmly increase the interval in minute steps. The beats will gradually get slower until they suddenly disappear altogether, the sound

* See chapter 13.

† When one tunes to perfect fourths, fifths and octaves, the exact Hz are 362.04 Hz; when tuning with open fourths, fifths and octaves, they are 362.40 Hz. But as the tuning forks are calibrated at ± 0.5 Hz the difference does not come into account. See chapter 13, pp. 60–3 , and especially step 4.

of the interval opens up and the differential begins to sound strongly and sonorously. Open fourths and fifths have an unmistakable clarity and width and an intimate warmth that is very pleasing. At the same time, the tones which encompass the interval sound as individual Beings meeting together in free interchange. These open fourths and fifths, together with their sonorous difference tones, lend their richness of colour to a scale of twelve fifths tuned in this way and strengthen its characteristic of simultaneously filling the space with sound.

Table 28 gives a musically notated tuning guide. The open note head of each interval indicates the tone from which one tunes, and the solid head the one to be tuned. The dotted diamond-shaped heads on the base stave show the differential of the interval directly above. Maria Schueppel's* suggestion of a new accidental in the form of an isosceles cross for the geometric mean tones (+), has been adopted. The balancing quality of these new tones is thus also shown in the notation. The three tones on the extreme left indicate the tones c^1 = 256 Hz, $gelis^1$ = 362.40 Hz and a^1 = 432 Hz. First tune these three tones as exactly as possible. They must equal the tone of the tuning fork so exactly that they are beat-free. This is best achieved when one inwardly experiences the tuning fork's tone and then recreates the experience on the instrument.

Tuning proceeds in the two directions of movement belonging to the Apollonian scales: the pre-Christian descending direction of the true Dorian octachord and the ascending direction of the true-tone C major resurrection scale (see Table 11 and chapter 19). Starting with c^1 = 256 Hz, tune an ascending open fourth to f^1 and open fifth to g^1. Then tune a descending open fifth from a^1 = 432 Hz to d^1 and an open fourth to e^1. From this e^1, tune a descending open fourth to b and an ascending open fifth to b^1. The seven non-altered diatonic fifth tones of the scale of twelve fifths have now been tuned. The first open octave sounds between b and b^1.

As the two directions of movement meet in the fourth d^1–g^1, the first steps must be carefully tuned. The first formed fifth appears between b and $gelis^1$ (see chapter 13). It sounds serious, but it must be calm and totally harmonically acceptable. If this is not the case, look first for the cause of the problem in the b which has probably been tuned too sharp, and not in the $gelis^1$ which has been tuned from the tuning fork. Then go back to the descending open fourth a^1–e^1 and check this thoroughly. Pay attention to tuning the e^1 really flat enough. This fourth is wide and open and is correctly tuned when its differential sounds really strong and sonorous like the sound of an organ. The same applies to the descending fourth e^1–b. When these two fourths are tuned correctly, then the formed fifth b–$gelis^1$ also sounds correct.

Now proceed from $gelis^1$ and tune a descending open fourth of the same size to $delis^1$, and from there an ascending open fifth to $alis^1$; from $alis^1$ a descending open fourth to $elis^1$ and from $elis^1$ an ascending fifth to $belis^1$. The second formed fifth sounds between $belis^1$ and f^1. What applied above is also applicable here: if the formed fifth $belis^1$–f^1 sounds wrong, go back to $gelis^1$ and tune the tone sequence $delis^1$, $alis^1$, $elis^1$, $belis^1$ once again until they are correct and the formed fifth belis–f^1 sounds right.

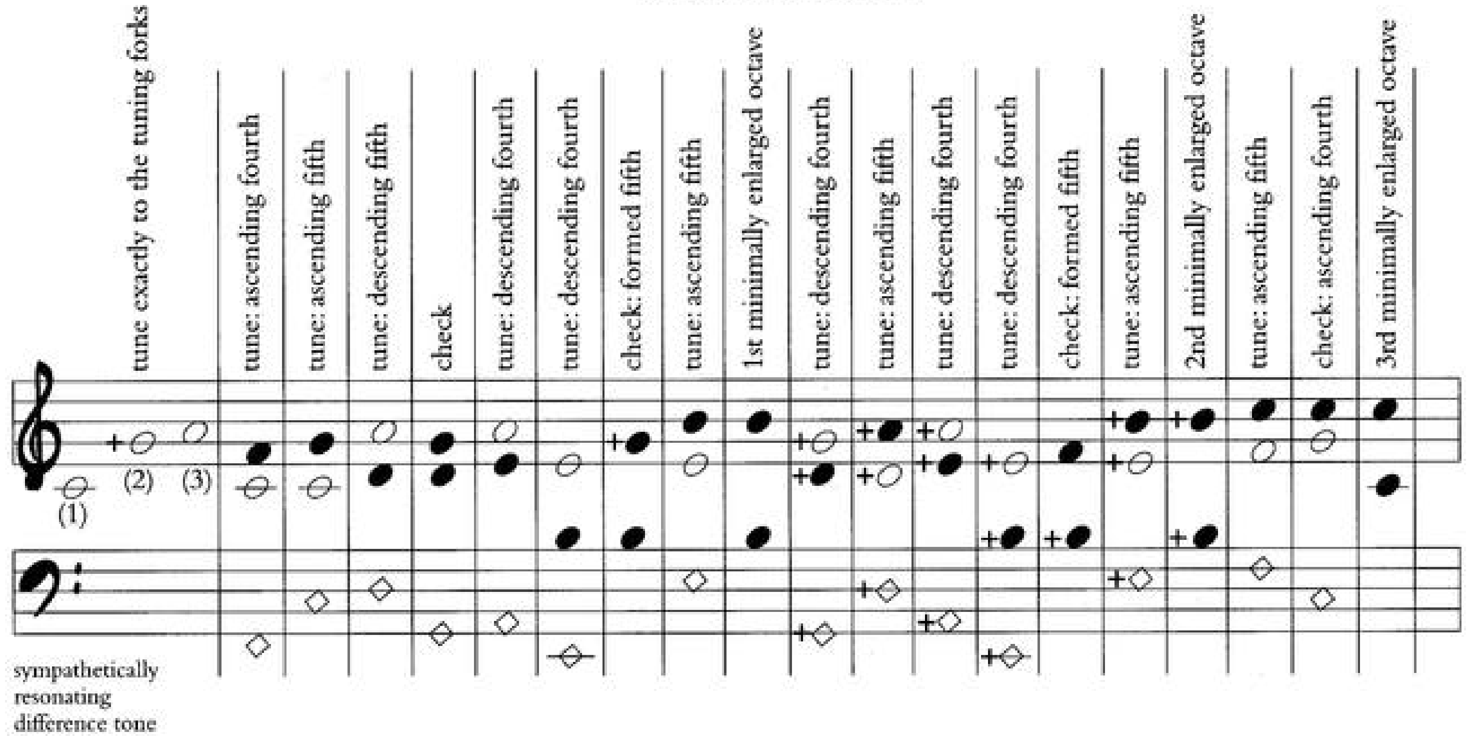
(The geometric mean tones can also be tuned as follows. First tune $gelis^1$ to the tuning fork then play c^1 followed by d^1 and tune $delis^1$ so that it is the exact tonal centre between these two tones. The descending open fourth $gelis^1$–$delis^1$ should then sound right. Proceed similarly for $alis^1$. First play g^1, then a^1 and now tune $alis^1$ as the exact tonal centre between them. The ascending fifth $delis^1$–$alis^1$ should sound correct. With regard to the descending open fourths—$alis^1$–$elis^1$ and $elis^1$–$belis^1$—care must be taken that these are tuned large enough, as was shown above.)

Finally, tune the two ascending open fifths $elis^1$–$belis^1$ and f^1–c^2 and check the latter with the open fourth g^1–c^2 (Table 28). The 9th belis–c^2 has thus been tuned to the scale of twelve fifths and the three minimally enlarged octaves b–b^1, belis–$belis^1$ and c^1– c^2 have been gained.

* The composer Maria Schueppel was the leader and founder of the 'Musiktherapeutische Arbeitsstaette' in Berlin.

Table 28
Tuning a keyboard instrument or lyre to the scale of twelve fifths

Temperament octave

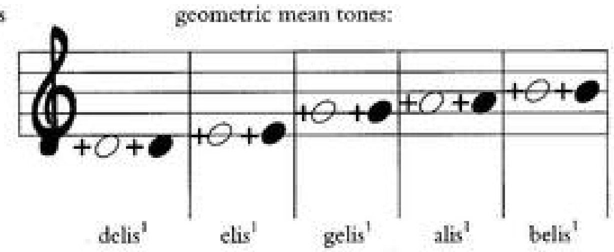


○ = tone from which tuning proceeds

● = tone to be tuned

◇ = sounding difference tone

geometric mean tones:



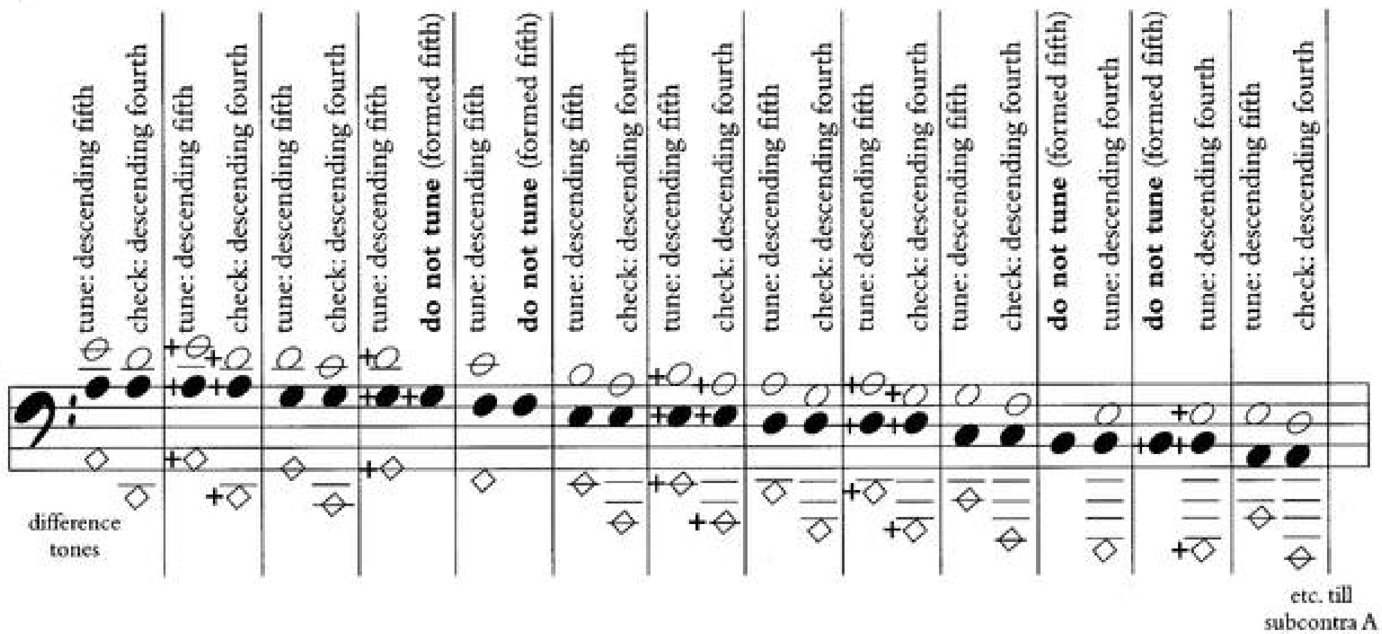
tuning forks at the following frequencies are required for the tuning:

(1) $c^1 = 256$ Hz

(2) $gelis^1 = 362.40$ Hz

(3) $a^1 = 432$ Hz

Table 29
Tuning the descending octaves



NB: only tune in fourths and fifths, never in octaves because all octaves are minimally larger than 2:1

Important: after tuning from c^2 to A play all 24 arpeggios in ascending direction. Every one must sound clear, beautiful and harmonically usable. None may sound harsh or rough

Here it must again be pointed out that tuning the scale of twelve fifths with the help of a tuner can only give an approximation of the reality. At best it can therefore only be a guide but never take the place of the ear. The reality can only be found (e.g. afterwards) by ear.

Tuning then proceeds with the lower octaves (Table 29). Start with e^1 and tune a as the lower open fifth—the difference tone A must be sonorous. Check the pitch of a by checking the interval to its upper fourth d^1. The difference tone D must be strong and sonorous. It is advisable always to check each newly tuned tone against its higher open fourth; this provides a stable base for the further tuning

Success in tuning the lower octaves downwards is again dependent on tuning the open fourths and fifths large enough, i.e., tuning the lower tone flat enough. To make sure that tuning has been successful, the following check is imperative. Having completed the lower octaves between c^2 and subcontra $_2A$, play all 24 major and minor arpeggios in ascending order one after the other. If they sound pleasant to the ear they have been correctly tuned. If each higher octave sounds too flat, then the open fourths and fifths have been tuned too small, with each new tone then tuned too sharp. In this case, return to e^1 and tune again until the correct size has been achieved and all chords sound pleasing.

To tune the higher octaves of the instrument, proceed from $gelis^1$ and tune in ascending direction to open fourths and fifths as shown in Table 30. A similar problem exists here as above, but the opposite. Take care that each new tone is tuned sharp enough, in other words that the open fourths and fifths are really tuned open. The sonorous resonating of the differential can be a helpful guide. When tuning is complete, play all 24 major and minor arpeggios again in ascending direction through all registers. The incorrectly tuned tones will be immediately and uncomfortably noticeable. When they have been corrected and all chords sound pleasing, the instrument is ready to be played; you can play in all styles and rejoice in a true and beautiful sound.

So there are now two methods of tuning an instrument of fixed tuning to aurally genuine, melodically and harmonically usable intervals to have tones that leave the human being totally free and nevertheless also belong to him. The first method, made up of beat-free fourths, fifths and octaves, has a sturdy, nice, rather silvery character; the second with its open fourths and fifths and minimally enlarged octaves has clarity, breadth and warmth. The human being can use both to enrich his music making and develop a deeper relationship to intervals, scales and tones.

The early Greek aulos modes rediscovered by Kathleen Schlesinger (chapters 8, 9 and 18) can also benefit in richness of sound and beauty when tuned to minimally enlarged octaves.

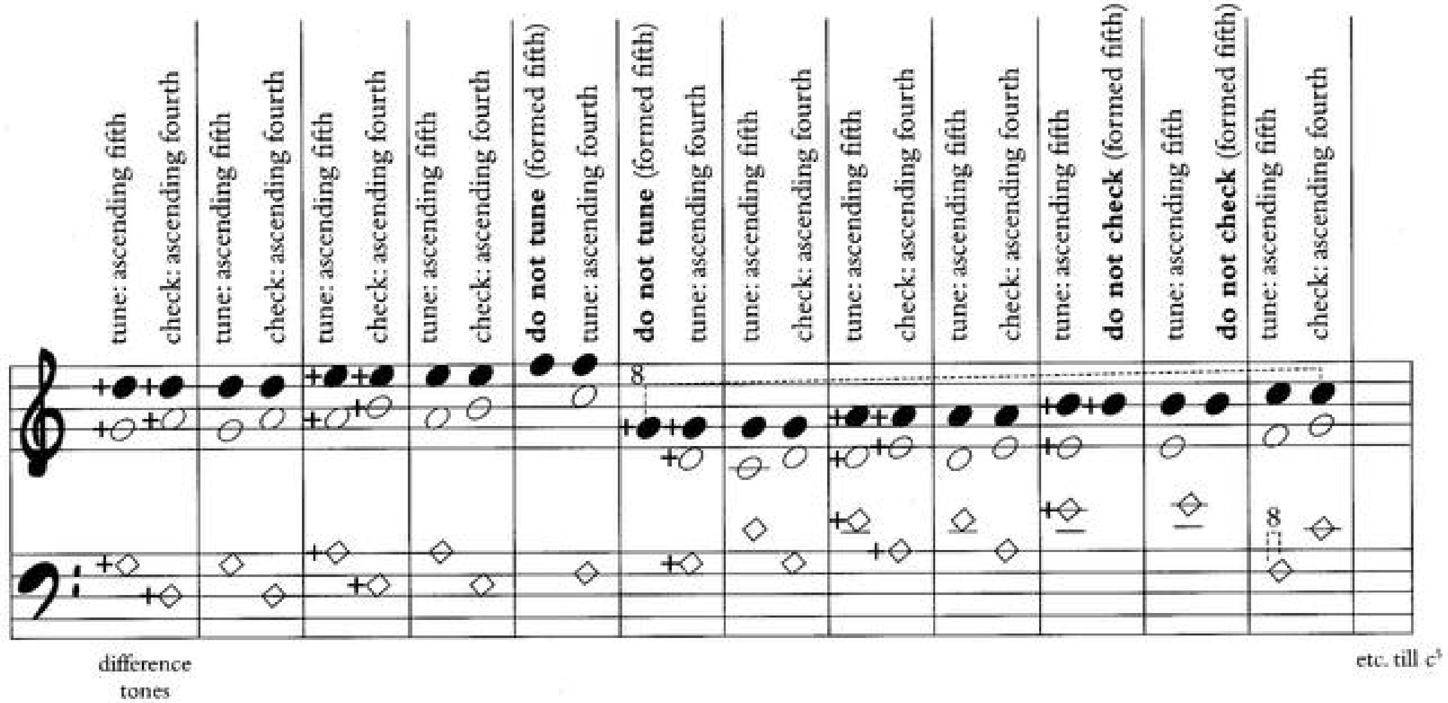
The descriptions given by two professional tuners follow below. Lothar Thomma, master piano builder and constructer at Bechstein, and Thomas Henke, concert technician and chief intoner of the firm Rud. Ibach Sohn, have both tuned the Bechstein concert grand piano specially lent to the Goetheanum for tuning the scale of twelve fifths. They have kindly put their experience and observations, as well as further suggestions for the tuning, on paper.

Lothar Thomma

Twelve fifth-tones tuning according to Maria Renold

The greatest hurdle when tuning the scale of twelve fifths are the first intervals. The tones that are given for this tuning are c^1, a^1 and $gelis^1$. Except for two fifths, ten fifths in the circle of fifths are perfect. This displacement, as well as for example the true thirds and sixths, which, compared to equal-tempered tuning beat very quickly, require very careful work during the first steps, as sketched in Table 31. Comparisons back and forth, together with the necessary corrections, ensure a result that meets the requirements of the 'discoverer'. The method depends even more than the normal equal-tempered tuning on the proportions of the total acoustic arrangement. It is therefore necessary to pay continuous and observant heed to the fourths and fifths during the practical work. As the fourths and fifths have such special value, the octave steps, usually regarded as the perfect proportion, lose their rank of first place.

Table 30
Tuning the ascending octaves

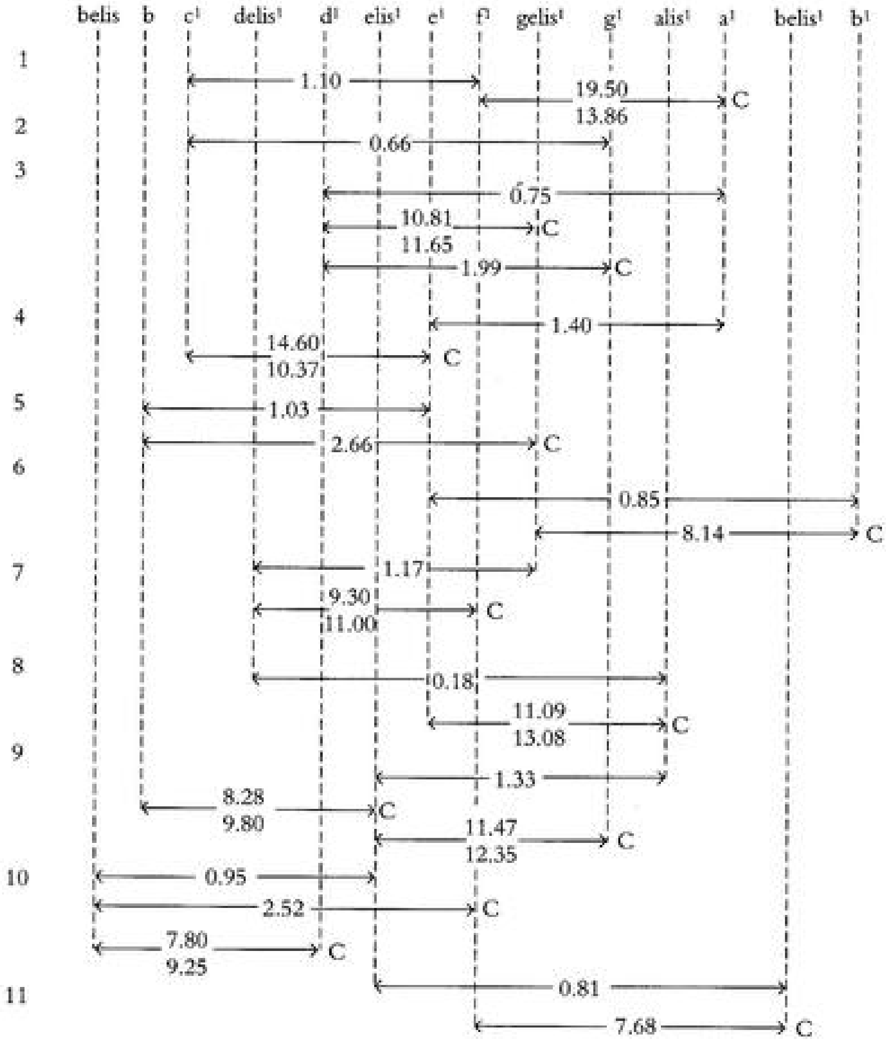


NB: only tune in fourths and fifths, never in octaves because all octaves are minimally larger than 2:1

Important: after tuning the ascending octaves, play all 24 arpeggios in ascending direction through the whole range from A to c^5. Each one must sound clear, beautiful and harmonically usable.

Table 31
Lothar Thomma: Tuning the scale of twelve fifths according to Maria Renold

Given pitches: c^1 = 256.00 Hz, $gelis^1$ = 362.40 Hz, a^1 = 432 Hz
c^1–d^1 = 16.00 beats; normal = 11.85



Intervals with two numbers: upper number = required beats, lower number = equal temperament. C = check

It needs to be especially pointed out that all indications of beats in Table 31 are made without taking the inharmonicity into account. The stated sizes can therefore only be taken as an indication.

Thomas Henke

Instructions for and experiences with tuning the scale of twelve fifths according to Maria Renold

In the first instance a new method of tuning makes the piano tuner sceptical, since it is almost taboo to change equal-tempered tuning which has been in use for nearly 250 years and it certainly seems presumptuous to wish to improve on it.

It is a challenge and an unusual approach for a piano tuner who commits himself to tuning the scale of twelve fifths, mainly because one has no tempered octaves and thus no corresponding control. This method of tuning has been developed from a different way of arranging the twelve minor seconds on a polyphonic instrument. The first step towards coming to an understanding can thus only be to establish why Maria Renold has sought and found this method.

As a violist, Maria Renold particularly experienced the discrepancy that arises for the ear when the method of tuning and the intervals of a stringed instrument are compared with the same tones and intervals of a keyboard instrument. The deviations can be clearly audible. This is because string players have true tuning (based on tuning in perfect fifths) whilst the piano's tempered tuning is a compromise.

The first fundamental difference is that there are no perfect octaves—the octave is the only interval in tempered tuning which is perfect—whereas, in keeping with the tuning of string players, the fourths and fifths are perfect and open. The octaves will therefore always be larger than perfect! As piano tuners we are tainted because we always tune and hear fifths and fourths that are not perfect but tempered. It is therefore essential for tuners who have difficulties with this to be in touch with string players, singers and choir conductors with an excellent ear for perfect and open tuning, who can demonstrate, by playing and singing, how rich perfect and open intervals can sound. A tuning experiment with perfect and open fifths may even let a not so musically trained tuner quickly hear how the fifth increases in richness of sound when it is widened from tempered to perfect and to open. It then sounds clear, straight and bright because of the resonating differential (lower octave of the prime), and also warm and sonorous.

The difference tone also makes it necessary, for a larger fifth to be sometimes tuned over and beyond the perfect fifth, so that the difference tone and its quality is clearly heard.

The three specified tuning forks initially make no sense to conventional tuners. But even the first attempts at tuning will make it clear to the beginner that one should not diverge from the narrow path through the temperament octave in even the least degree if a useful result is to be obtained. The end result allows no compromise! The temperament octave will either sound out—or offend the ear. With tolerance much less than in a normal tuning, fixing the three tones is crucial. The first step is therefore a very exact tuning of the three tuning fork tones. It is no longer sufficient to achieve an apparent similarity of sound between tuning fork and piano. A tone can be bright, shining, dark, covered, warm or cold. It is very important to follow how a tone changes its sound quality when ringing out and dying away, and to go so far as to attempt to call it a red, blue, brown tone, etc.—in so far as one actually has a feeling for this. (This also brings us to a further peculiarity, namely, a verbal description of what we hear. It will therefore be necessary to get in the habit of using words never used before to describe the quality of an interval or of a tone.)

I expressly recommend listening to the tuning fork tone for a long time, and repeatedly sounding it again several times until one has made it one's own, and only then playing the relevant tone on the piano. The point is not so much to express the nuances in the pitch but to determine them by the feeling of the sensory experience indicated above.

Having tuned the three tuning fork tones, tune the fourth and fifth upwards from c^1 and downwards from a^1 wide and open with the fullness of the differential. The white keys between c^1 and a^1 are fixed in this way. The interval d^1–g^1 results as a matter of course and is minimally larger than perfect. Thereafter the fourth e^1–b is tuned downwards with the tuning tone $gelis^1$ making it possible to check. Maria Renold calls the b–$gelis^1$ interval a formed fifth. The difference is clearly noticeable after the many open fifths. Tuners like to use beats to help with tuning, and, with reservations, I would say use about three beats for the open fifth. It is clearly more than tempered and shortly before the point when it sounds sharp and no longer harmonic.

Both formed fifths in the tuning octave, b–$gelis^1$ and belis–f^1, mark the transition from white to black keys. They need to be tuned with care and require special attention when tuning in the bass. As both the black and white groups of tones are in themselves tuned to open fifths, the formed fifths are solely responsible for the quality of the whole tuning, because of the relationship between the white and black tones. The minor second is not only exactly in the geometric middle between two white tones but also in the geometric middle of the octave belonging to it; $gelis^1$ is therefore the geometric middle between f^1 and g^1 and also between c^1 and c^2. A feeling needs to be developed for $gelis^1$ as a tone that is in the geometric middle. The exact geometric middle is evident in Table 32. Played together, we can hear how different the black mean tones sound to those with which we are familiar. If we imagine a harmonic progression from D to G ($f\sharp^1$ to g^1), we have the f $sharp^1$ somewhat sharper and brighter in our ears—as we know it from many recordings and our equal-tempered tuning (true third in Table 32). If we play the G major chord and imagine an $f\sharp^1$ in D major until we think we know how it should sound, and then play $gelis^1$, as the exact centre between f^1 and g^1, a slightly flatter tuned tone is heard. I wish to call this minor second a 'muted cheerfulness'. This feeling comes with all black geometric mean-tones.

Having got used to $gelis^1$ as a middle-tone, tune the remaining minor seconds from there with open fifths and fourths.

Maria Renold lays great importance on now testing all triad combinations within the central octave. No tone should sound rough and shrill.

Having tuned the temperament octave, move on to the bass and then the soprano. The inner tones of the formed fifths (B and F) may only be tuned from fourths. The interval steps for the first bass and soprano octaves are given below.

First temperament octave in the bass

alis	←	$elis^1$	fifth
g	←	c^1	fourth
gelis	←	$delis^1$	fifth
f	←	c^1	fifth
e	←	a	fourth
elis	←	belis	fifth
d	←	a	fifth
delis	←	alis	fifth
c	←	g	fifth
B	←	e	fourth
Belis	←	elis	fourth
A	←	e	fifth

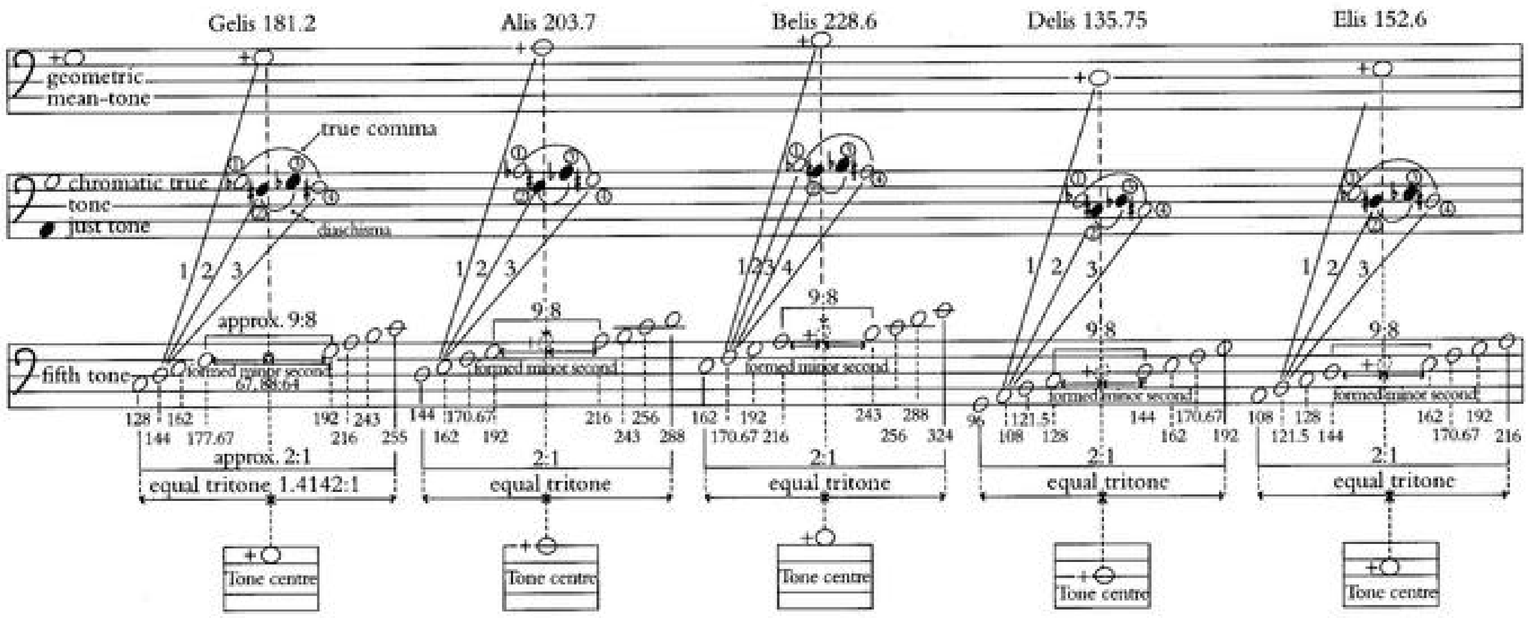
First temperament octave in the soprano

$elis^1$	→	$belis^1$	fifth
e^1	→	b^1	fifth
f^1	→	c^2	fifth
$gelis^1$	→	$delis^2$	fifth
a^1	→	d^2	fourth
$belis^1$	→	$elis^2$	fourth
a^1	→	e^2	fifth
c^2	→	f^2	fourth
$delis^2$	→	$gelis^2$	fourth
c^2	→	g^2	fifth
$delis^2$	→	$alis^2$	fifth
d^2	→	a^2	fifth

The question as to the importance of Maria Renold's requirement to tune only on the basis of c^1 = 256 Hz and a^1 = 432 Hz should not be rejected out of hand. Apart from anything else, respect for the comprehensive work in this book demands that one at least considers the tonal image of the pitch and takes note of the relaxing dynamics of music played on it.

Table 32

The proportion of the geometric mean-tones to the neighbouring just and true tones



1 formed major third 80.45:64
2 just major third 5:4
3 true major third 81:64

1 formed fourth 4.07.3
2 perfect fifth 4:3
3 augmented just major third
4 augmented true major third

① G♭ 179.797 Hz ③ g♭ 182.044 Hz ① A♭ 202.272 Hz ③ a♭ 204.8 Hz ① B♭ 227.555 Hz ③ b♭ 230.4 Hz ① D♭ 134.848 Hz ③ d♭ 136.533 Hz ① E♭ 151.704 Hz ③ e♭ 153.600 Hz
② f♯ 180 Hz ④ F♯ 182.25 Hz ② g♯ 2002.5 Hz ④ G♯ 205.931 Hz ② a♯ 227.812 Hz ④ A♯ 230.660 Hz ② c♯ 135 Hz ④ C♯ 136.687 Hz ② d♯ 151.875 Hz ④ D♯ 153.773 Hz

In conclusion let us consider some secondary phenomena of the new tuning method. If one tunes the middle ninth (belis–c^2) on the basis of c^1 = 256 Hz, as indicated, $gelis^1$ = 362.40 Hz and a^1 = 432 Hz and then the further registers to open fourths and fifths, i.e., with the resulting octaves minimally enlarged one will obtain a higher tonal region, in the octave a^4–c^5 that bears the quality of tone a^1 = 440 Hz. Then, descending from the middle to the first and second subcontra octaves ($_1A$–$_2A$) one enters an area with the quality of just $b\flat^1$ = 448 Hz. The tones of the higher region have a strongly luciferic, those of the lower region a strongly ahrimanic quality (see chapter 21, pp. 124–5). These qualities are at home in these extreme registers, but only there. In the heights, where the tones almost dissipate, the inciting quality of a^1 = 440 Hz gives the tones independence. They are so much above the range of the human voice that they can hardly induce antisocial behaviour. In the depths, where the tones almost drown in a sea of noise, the hardening quality of just $b\flat^1$ = 448 Hz gives firmness. But these tones are so low that the human being will not come under their hardening influence.

Three spheres of activity around and in the human being, with all in-between stages, are thus mirrored. In the middle we have the free-leaving, healing power of the Christ; in the heights increasingly the seductively inciting power of Lucifer; in the depths, also increasingly so, the compressing and hardening power of Ahriman. So long as human beings connect with the Christ power of the middle through the tones between c = 128 Hz and a^1 = 432 Hz, they are able to include the powers in the other two registers in their music making.

Unpleasant experiences in recent years demand that a final reference is made to the fact that the scale of twelve fifths must and should only ever be tuned from *c^1 = 256 Hz, a^1 = 432 Hz and $gelis^1$ = 362.4 Hz*. The reason for this lies in the effect that the inherent qualities of other concert pitches have on the human being (see chapters 13, 15, 16, 18, 21 [p. 107] and 27).

Table 33

Tuning the twelve fifth-tones scale with an electronic tuner

Mean values, calculated from several tunings of the twelve fifth-tones scale on grand pianos, pianos and lyres, measured with a 'Langbein SQ tuning set'.

The numbers presented in the table give an approximately correct tuning; but because each instrument speaks differently, each tone should always be finely tuned by ear.

CONCERT PITCH: ROUGH ADJUSTMENT OF THE TUNER TO a^1 = 432 Hz

Basis: octave proportion = 1:2.003873803 = 1203.350 cents

semitone interval in cents	tone	cents from c^1	cents from a	Hz	PITCHES: FINE TUNING OF THE TUNER IN CENTS: equal tempered	octave:	twelve fifth-tones scale:**
	a	−297.485	0	215.6	a = 0	220	a = −3.485
101.775							
	belis	−195.710	+101.775	228.6	$b\flat = 0$		belis = −4.290
101.775							
	b	−93.935	+203.550	242.5	b = 0		b = +0.065
93.935							
	c^1*	0	+297.485	256.0*	$c\sharp^1 = 0$		c^1 = −6.000*
101.775							
	delis[1]	+101.775	+399.261	271.5	$c\sharp^1 = 0$		delis[1] = −4.225
100.640							
	d^1	+202.415	+499.900	287.8	$d^1 = 0$		d^1 = −3.585
101.775							
	elis[1]	+304.190	+601.675	305.2	$d\sharp^1 = 0$		elis[1] = −1.8000
101.775							
	e^1	+405.965	+703.450	323.7	$e^1 = 0$		e^1 = −0.035
93.935							
	f^1	+499.900	+797.386	341.7	$f^1 = 0$		f^1 = −6.100
101.775							
	gelis[1]*	+601.675	+899.161	362.4*	$f\sharp^1 = 0$		gelis[1] = −4.325*
101.775							
	g^1	+703.450	+1000.936	384.3	$g^1 = 0$		g^1 = −2.555
100.640							
	alis[1]	+804.090	+1101.575	407.4	$g\sharp^1 = 0$		alis[1] = −1.910
101.775							
	a^1*	+905.865	+1203.35	432*	$a^1 = 0$	440	a^1* = −0.135*
101.775							
	belis[1]	+1007.640	+1305.125	458.2	$b\flat^1 = 0$		belis[1] = +1.640
101.775							
	b^1	+1109.415	+1406.900	485.9	$b^1 = 0$		b^1 = +3.415
93.935							
	c^2	+1203.350	+1500.835	512.9	$c^2 = 0$		c^2 = −2.650

* predetermined tones (tuning fork)

** the 'Langbein SQ 22 tuning set' is set at 100 cents per semitone step. But as the semitones of the twelve fifth-tones scale are either bigger or smaller than 100 cents (see the first column of the table) the fine tuning for each semitone needs to be individually set according to the last column of the table.

Comments

1) It is important to take note that when tuning the octaves beyond the precise decimals given here the octaves may not be exactly 1:2, but a bit larger, as shown above: 1:2.003873803 (see also chapter 24). Therefore the tuning proceeds in minimally enlarged fourths and fifths according to the above table and *never* in octaves!

2) The cents make it easily possible to tune this scale to other concert pitches than tones given above (*). But this should *never be done*, because, as has already been presented in chapters 14 and after, pitches are the bearers of specific qualities, which have an influential bearing on human beings. Only on the given concert pitches do the intervals and tones of the scale of twelve fifths have a harmonious effect on human beings, and at the same time leaving them totally free.

3) The divisions of a monochord with a string length of 93 cm on the fundamental c = 128 Hz are given in Table 34.

25 Structure of the Newly Tuned Scale of Twelve Fifths*

The new method of tuning the scale of twelve fifths is almost exclusively based on open fourths, fifths and octaves. As these intervals are larger than perfect, they overstep the rigid laws of the just undertone and overtone rows and their form principles. In spite of this the human ear not only experiences the varied sizes of the open intervals to be musically acceptable, but also to be correct and not false like the intervals of equal-tempered tuning. If one attempts, however, to use open intervals as the building principle for a scale, proceeding from an initial tone in open fourths and fifths, the resulting imperfect and dissonant intervals are intolerable to the ear. To create an aurally acceptable scale from the open intervals one needs to make use of the two directions of movement that are available today in the Apollonian music stream—the pre-Christian descending direction of the true Dorian octachord and the ascending direction of the true C major scale (see chapter 24). These two directions of movement were used in the analysis of Plato's *Timaeus* and in finding the form principles that lie at the base of the first method of tuning the scale of twelve fifths (see chapter 13). We met them again in considering the indications Rudolf Steiner gave to Lewerenz. It was these two directions which first made an understanding of the indications possible (see chapter 19). They are now used as a structural element for tuning the scale of twelve fifths itself. This involves them for the first time directly in evolving a scale.

Taking these powerful directions in musical movement into consideration when tuning an instrument and expertly connecting the various sizes of open fourths, fifths and minimally enlarged octaves with the help of an exact and unbiased listening ear, a musically valid chromatic scale can be created that sounds freely in space.

To gain the whole fullness of sound that is possible, the size of each interval needs to be exactly tuned by ear. The physical nature of each instrument to be tuned is different. The size of every interval must therefore be adapted to the individual instrument by ear. This is clearly pointed to in the tuning directions given by Lothar Thomma and Thomas Henke in chapter 24. In chapter 23 it was said that variable interval sizes make the cent an expedient at best, until the tuner is able to aurally recognize the open quality of their different sizes. The individual quality of the single degrees of the scale then remains unchanged because the measurable difference between the perfect and open intervals is very small.

With regard to this second new method of tuning the scale of twelve fifths, the human ear proves to be an organ through which human beings who use it without prejudice and with exactness can penetrate more and more deeply and comprehensively into musical realities.

* See Table 36, and Graphs 1–4 for a graphical comparison between the intervals of the first and second method of tuning the scale of twelve fifths and other tunings.

26 Research on Pitch

Research relating to pitch can be found as far back as early Greece. Here the problem will be considered from the point of view of the modern ear as was begun in chapter 16. This will add some new aspects that have been discovered in the meantime, some of them having been published in the German-language weekly *Das Goetheanum*.[89]

By chance, the author discovered Werner Straub's dissertation on tone quality and pitch, written in 1928. This is particularly interesting for us as the tone experiments were not done with electronic instruments (as has become usual since the 1930s) but with an Appun's sonometer, a reed instrument that is hardly to be found in laboratories today. The tones are produced by air pressure, similarly to those of a harmonium, and are therefore of the same quality as tones that are sung or played on instruments. Straub reported that, without exception, two groups of 14 and 16 lay people respectively were able to perceive a difference of about an 8th of a tone in the quality of pitches. More recent clinical aural experiments hardly ever refer to such differences in quality. This may partly be due to the fact that present laboratory tests are almost exclusively done with electronically produced tones, in which the quality of the tones is extinguished (see chapter 23), and partly to the fact that quality is not measurable and therefore not taken into account.

This human ability, also described by Straub, to aurally recognize and describe a difference in quality between closely neighbouring pitches is found among people of all strata of society (confirmed for Europe and America in the listening experiments described in Part Two of this work). Because of this subtly differentiated feeling for tone in modern people, it has become necessary and possible to demonstrate c = 128 Hz as concert pitch and the harmful quality of other nearby chamber pitches in public, even though the latter, for example a^1 = 440 Hz and c = 130 Hz or still higher chamber pitches, enjoy great popularity at present. Dietrich Marx, proponent of the new string instruments built according to indications given by Rudolf Steiner, continually points to this problem in his extensive lecturing and demonstration tours. He has been able to achieve a certain amount of success in Rudolf Steiner Schools and with musicians and eurythmists. Thanks are given here for his efforts, for the world of music tends to be rather conservative today, holding fast to old, habitual and well-worn traditions.

It has therefore given particular pleasure to the author that Elisabeth Ahlers, trained as an organ builder and piano tuner, who has contributed much to our results, has written quite a long report in which she speaks of how she sees the problem of pitches and their differences in quality. She writes:[85]

> Maria Renold maintains that tones connected with the concert pitch c = 128 Hz have special qualities, which human beings can experience directly. The tone c at 128 Hz is supposed to have a quality that makes it the fitting concert pitch for the present state of human consciousness. Having worked professionally as a piano tuner, I was not able to see anything other than a mere conjecture in this statement. But, as suggested in the introduction, a simple thought nevertheless led to the conviction that it might be worth while to test the truth of this statement. If tones actually did have individual qualities, and I did not notice this, I would all the time be dealing with something of which I had no idea. I might thus fare the same way as someone who carelessly handled pitchblende or carelessly threw away a golden hammer.
>
> One thing can be said right away. Frequency numbers tell us nothing of whether a pitch has individual qualities or not.
>
> How does one investigate a tone? To find a procedure that would do justice to the tone I had to find different ways to those used by physicists. The experiments, which I will describe below, have certainly only been a humble beginning. But it is possible that they may encourage readers to take up the subject or at least to develop an understanding for the method and content of such investigations.

A tuning fork for c^1 = 256 Hz is provided with the book that had proved a stumbling block for me. First of all concerning myself with the tone itself, for weeks I listened to the tone on the tuning fork for at least five minutes a day. Initially the greatest problem was to really listen, for in the attempt to listen to the tone I felt as if the ear found nothing to get hold of.

Something remained when the tone had faded away; I noted a change which I would like to express in a metaphor. Someone has said something important to me, and I cannot remember what it was. Nothing, but absolutely nothing in the room holds a trace of what has just been. A particular sensation arose when I realized that a friend had left. It reminded me of the feeling I had when hearing the tone. At first I did not venture to consider this observation to be a special characteristic of the tone c^1, as it was the only tone to which I was devoting myself in this way. I distrusted the exclusive procedure.

The ideal condition for unprejudiced judgement is given when one does not know in advance which tone is going to be played. When for the first time I did not play the tones myself but had them played for me on a monochord, I no longer felt concerned that I would merely suggest the sensations for myself. I could enter into the tone, without my prejudices being in the way. However, the experience was still faint and subtle. I was only able to find a difference between the tones at a level where I did not usually make observations. When I asked my feelings, sympathy and antipathy had really nothing to say. The questions 'Do I like the tone?' 'Is the tone beautiful?' did not lead anywhere. The sensations evoked by different tones actually arise where I imitate basic attitudes of soul that enable me to observe. The first rough images I found for the three c-tones may illustrate this. The three tones were

c = 126.2 Hz (corresponds to a^1 = 424.5 Hz)
c = 128.0 Hz (corresponds to a^1 = 432 Hz)
c = 130.8 Hz (corresponds to a^1 = 440 Hz)

With c = 126.2 Hz, a sensation arose for which I had the image of someone lying down to rest without having earned it, of sitting down in an armchair with a gesture of never wanting to get up again. The c = 130.8 Hz plainly and simply agitated me. With the c = 128.0 Hz I experienced, in direct comparison, neutrality: I could not be forced to sit in an armchair nor to make an effort. Perhaps it is not so easy to see immediately why tones that are so close together in pitch should have such opposite characteristics. For the time being, I would like to support this observation with a passage from Goethe's *Theory of Colour* (Part Six. Sensual and moral effects of the colours. Yellow): 'When the colour yellow is impure and is imparted onto a base surface such as a common cloth, felt and suchlike, whereon it does not appear with all its energy, such an uncomfortable effect arises. By means of a minute and unnoticeable change the beautiful impression of fire and gold is changed into the sensation of excrement and the colour of honour and delight is turned into the colour of shame, abhorrence and displeasure.'

Another exercise, which I did almost at the beginning, was in a way secondary to my attempts at observing tone sensations. If the tones have individual qualities, then it must be possible to produce a specific tone inwardly without having absolute pitch, as it is called. I therefore attempted to hear c inwardly. At first I did not know how and where I should look for it. A tone would sometimes appear, but it was variable and I remained uncertain. Now and then, I did perceive a tone, faintly so, but quite definitely and certainly. I immediately tuned the C string of the harpsichord to this tone and then compared it with the tuning fork. Every time I had tuned exactly c^1 = 256 Hz. It was evidently the inner activity which mattered. Either I am able to get myself into the state of mind where the tone can arise and then I find it exactly and unmistakably, or I do not penetrate to the appropriate area at all. To put the situation in two metaphors again, when someone is supposed to buy onions and has no success, this may be due to several things. Onions are not available, the shop is closed, the person had taken no money with him. The failure may thus also be due to things other than the purchaser.

The situation is certainly different if I want to watch the stars. They are actually always there, but you often cannot see them. This second metaphor can be related to the tones, if one compares the weather situation to one's own state of soul.

There was need to observe which activity I might use to produce a specific tone. Once again, I began to listen to c^1 = 256 Hz on the tuning fork, entering wholly into the tone. I started as if I wanted to sing the tone and went more and more in the direction of totally 'being' tone. Something then arose that was as if the tone continued to sound. The fading away of the tuning fork sound was wholly obliterated by intensive inner listening. The inner gesture I thus created can also be achieved without the inducement of hearing the tone. Here lies the key to hearing in advance. It is not a matter of inwardly forming the idea of the tone, as I imagine a violist would do before playing an open C string, in order to expect what will then be heard. Instead I make an inner movement in complete inner clarity and thoughtfulness in which the tone can then 'sound'. It very much depends on my inner attitude when listening if the tone can appear or not. I must be totally aware of what I am trying to do and banish all other thoughts.

When I listen in such peaceful expectancy, an initially silent sensation arises, which then condenses to a resounding tone. For c^1 = 256 Hz I would describe the process as follows. I first silence everything in me that relates to my subjective condition and inner mood of the moment. At the same time I pay attention to the sensation that arises as I visualize, with the greatest possible openness and honesty, my existence as a complete human being. I then listen for the tone that can arise in me as a complete human being, without drawing me under its spell. When it becomes inwardly audible this tone proves to be c^1 = 256 Hz or its lower octave at 128 Hz.

A minor hurdle arises with hearing in advance because one attempts to force the evolving tone too quickly into the clear idea of a tone. I found over and over again that the tone would become clearly perceptible. I then sent ideas of tones towards it, as it were, and always put my money on the wrong horse in thinking one of them to be the right one. It was exactly by thus losing my way that it became evident that I only produce the tone in so far as I help it to arise and not try to generate it. Another problem would play tricks on me when I wanted to use an external instrument to check which tone I had found. If I attempted to find the tone a^1 and held the a^1 = 440 Hz tuning fork to my ear, it would often be only the first small moment of estrangement that would tell me that the outer did not correspond to the inner tone. The ear would instantly take up the outer tone as if it had already a set place at this point.

How should I form a concept of the tone so that it will not be destroyed by such observations? The essential nature of the tone evidently does not arise solely because neural impulses are produced in the human ear by means of regular vibrations of a body of air waves, or electronic impulses in a microphone. Tone is a real entity, extant and independent of such phenomena, making itself known directly to the human soul. The physical or physiological phenomena observed with tone are necessary if it is to reveal itself in the sense-perceptible world. In the answers to questions during the first science course, we find the following statement made by Rudolf Steiner:[45] 'Neither tone nor warmth, nor light, nor electricity are waves (beats), as little as a horse is the sum of its gallops. Tone, for example, is a quality with its own essential nature, and the effect of its progress through the air are waves or beats. For the sentient human being, the waves are the inducement to imitate this quality within himself: that is how tone is perceived.'

Recently the composer and eurythmist Bevis Stevens investigated how the tones c = 128 Hz and c = 130.828 Hz as well as equal-tempered tuning and twelve fifth-tones tuning, in comparison and alone, are experienced by the eurythmically moving human being and what effect they have on someone watching.

Bevis Stevens

Experiences in doing eurythmy to a piano tuned to the scale of twelve fifths at the concert pitch c = 128 Hz compared with a piano tuned to equal-tempered tuning at c = 130.828 Hz*

In repeated experiments, the concert pitches c = 128 Hz and c = 130.828 Hz and equal-tempered and twelve fifth-tones tuning have been eurythmically compared with one another, singly and in groups.

The tone c = 128 Hz as prime streams evenly and harmoniously through the whole stature. Even the feet are self-evidently 'there' and can be reached without exertion. The eurythmist rests and streams within and through his whole body. With the tone c = 130.828 Hz the eurythmist experiences himself to be drawn upwards, away from the feet. The chest and head area expanding, the feeling is as if he is blown up with air which he is holding (this is to be understood in the sense of the eurythmy figures), because the tone does not sound through but outside of the body.

The same fundamental tendencies appear with the 30° tone movements. The tones relating to c = 128 Hz rest within the area of the muscle and bone structure of the human being. With c = 130.828 Hz the eurythmist can imagine these movements to lie within the body but they never become real and direct experience there. If the eurythmist tries to find where they lie, the place, and/or angle cannot be found; he experiences only that they lie 'outside'.

By eurythmical improvisation to the C major prelude from J. S. Bach's *Well-tempered Clavier*, the following experiences arose. With c = 130.828 and equal-tempered tuning one experienced a world of most beautiful radiance. Nothing more beautiful exists. In striving after this 'world' the eurythmist initially feels bigger, like a 'wonderful soloist'. It is a great feeling! But it deludes one; this 'world' of most beautiful radiance is outside of him and his true being.† It proves to be an unattainable illusion, and the human being loses his humanity, his freedom. If the attempt is made to bring this world of beautiful radiance to expression—and this is only possible in that the eurythmist strives after this world, because it does not come to him—then he tires very quickly. For the observer, the eurythmist looks tense and small and the movement angular. This is due to the strong muscle tension which is induced. Pushing this muscle tension to the extreme I have often had to stop going any further because of being gripped by a strong, fearful feeling of being about to lose myself. A group of eurythmists move very inharmoniously together.

* This account has been added to and revised for the English translation, as further work and observations have been made since the third German edition of the book (Bevis Stevens).

† This feeling of the music being 'outside' is most probably the reason why more and more concertgoers feel removed from the music and go home feeling empty. The music does not touch or move them. Only the greatest musicians are able to overcome this problem through inner activity, just as the eurythmist is able to ignore the actual quality of the tones and music, and due to a training that teaches him to work with the primal qualities and forces behind and in music, is able to create these within himself without having them outwardly present. Many people have responded by saying that it is exactly this that makes equal-tempered tuning at c = 130.828 Hz modern, as this leaves one free. However, I have often observed that especially 'unmusical' eurythmy students do the gestures wrongly in following the urging of their teachers to bring to expression what they hear, cannot understand why Rudolf Steiner described the gestures the way he did and finally put this down to being unmusical. My own experience has always, even before getting to know the twelve fifth-tones tuning and c = 128 Hz, been one of longing to find tones that really fit the experience of the 30° tone angles and the interval gestures (for interval gestures, see the end of this account and the footnote on p. 158). And why should one use something inferior and unfitting once something better has been found?

With the twelve fifth-tones tuning on c = 128 Hz the eurythmist feels as if he is being invited to move. He moves his arms and the world streams in. At the same time the boundaries of space open up and come alive. Relationships arise between world spaces. The eurythmist feels himself in balance between his centre and the world periphery. The connection between microcosm and macrocosm, between the human and the godly arises anew. The space between the arms is also filled, bringing about relationship and conversation between the arms. The arms find a connection to one another. The eurythmist does not need to strive after something. Rather he is left free to approach the music with questioning interest. He then receives an answer. He experiences a grace-given peace, peace as joy and quiet at the same time. Seen by the viewer, the movement is peaceful, but big, and the periphery is filled. A group of eurythmists move together harmoniously as if within a common form that envelops them. In eurythmy terms one would say that the etheric surrounding and streaming is full and evident.

Both pianos were in the same practice room, so comparisons could easily be made. Sometimes a period of adjustment from one piano to the next was necessary—after the brilliance and tension of equal-tempered tuning and c = 130.828 Hz (a = 440 Hz), the twelve fifth-tones tuning and c = 128 Hz could occasionally feel empty. Here the above-mentioned feeling of questioning interest was the key to finding the new quality. On other occasions the change was experienced as a relief. A change the other way around was mostly experienced as being unpleasant. On one day we would begin with the one piano and then we progressed to the other; the next day we started with the other one. Sometimes we stayed with one piano for the whole session. The most crass experience was made when, having grown sufficiently familiar with the differences so that we knew what to expect in advance, there was a misunderstanding and the group began the movement as if for equal-tempered tuning only to be met with twelve fifth-tones tuning. The surprise was great and even though they reacted immediately and corrected the movement, never before was the difference so clear. On another occasion the pianist told how the one piano required to be played very differently to the other and suggested that she could try to swap the difference around, playing on the one as if for the other. The effect was immediate. Playing on twelve fifth-tones tuning as if for equal-tempered tuning, the falseness was clearly observed. The feeling was like having caught a thief red-handed. The other way round gave the feeling that what was wanting to be musically created was being attacked and destroyed by the equal-tempered tuning, or, seen from the other angle, just did not speak—like someone who forgot to put salt in the meal.

A piano tuned to a pitch correspondingly lower than c = 128 Hz as c = 130.828 Hz is higher (c = 126 Hz) was not available, and comparisons were therefore made with tuning forks and tones played on a monochord. With the tone c = 126 Hz as prime, the eurythmist feels as if he is pressed so far into his body that the air is squeezed out of him. The head area feels very small and the inner movement weighs downwards. One is drawn through the feet into weight. (This is to be understood in the sense of the eurythmy figures.)

It has been found that the interval gestures or, more exactly, the streaming through the collarbone into the bones of the arms for the intervals, do not fit true intonation and twelve fifth-tones tuning. Investigation found a beautiful correspondence with the just intervals. It was therefore a joy to have this confirmed by an indication given by Rudolf Steiner during the teachers' conference of the eurythmy training where he gave the proportion of the intervals in relation to the proportion of the arm bones with the decisive interval, the third, as the proportion 5:4 which is the just major third.*

* *Die Entstehung und Entwickelung der Eurythmie*, GA 277a, p. 141.[18]

The quality of this third is truly that with which Rudolf Steiner described the third in the tone eurythmy course (*Music as Visible Singing*[33]) as 'the soul speaks with itself'.*

It is hoped that these reports by Elisabeth Ahlers and Bevis Stevens will contribute to bringing readers closer to the significance and import of the choice of chamber pitch for music making, and spur readers on to do their own research on the inherent qualities of tones.

* Much more could be said, but this would exceed the bounds and task of this book. However, it may be of interest for fellow eurythmists to know that research is now being carried out to find out if the *tone* gestures (as opposed to the 30° angle movements and their correspondence with true intonation) are related to the *tones* of just intonation and if the interval *gestures* (as opposed to the streaming through the bones in connection to the just intervals) are connected with the *intervals* of true intonation.

27 Absolute Pitch and c = 128 Hz

Absolute pitch, which is commonly seen to be an advantage, can prove a hindrance when changing to the concert pitch c = 128 Hz and a^1 = 432 Hz which belongs to it. Because of this, musicians have told the author that they felt bound by experience to the qualities of those tones which have become part of them, and that other tones, including closely neighbouring ones, sounded out of tune and false to them.

That such memorized pitches are by no means 'the only right ones' is shown by the fact that the hearing of people who are gifted with this often inherited ability of memory can relate to different pitches. The author knows three important musicians with absolute pitches at a^1 = 435, 440 and 444–446 Hz; to play chamber music together would be extremely difficult for them. On top of this comes the historical fact that from the eleventh century concert pitch has varied within the interval of a sixth (!), according to Ellis,[14] and, as the above example shows, is by no means stable today.

A solid foundation for concerning oneself with the problem of absolute pitch is given in the work of A. Bachem,[86] who, having this ability himself, investigated it in many people in America for a number of decades. Bachem mainly concentrated on the question of how absolute pitch arose, and how someone with that gift would recognize the tones. Yet he also considered whether it was possible to overcome the faculty when it was set to specific tone pitches. Two of many exceptional musicians with absolute pitch were found who were prepared to comment on this question from their own experience. Both have kindly given permission for their answers to be printed here. Christian Ginat, viola, gives concerts all over Europe and teaches in Dornach; the composer, concert cellist and flautist Gotthard Killian is mainly working in the field of education.

Gotthard Killian writes:

Absolute pitch is a kind of memory faculty that people will not easily set aside in favour of direct hearing activity, as they believe it to be of special value. However, it will only be possible to pursue and generate new qualities from the audible to the inaudible, and hence their essential musical value, if perception of tones is wide awake and not bound to memory.

The crucial point is the extent to which one succeeds in not only labelling and intellectually evaluating a phenomenon but at the same time in letting it speak out of itself. This ability is found to be more exact than any mechanical apparatus in the musical instrument from which we, as Goethe also found, can expect the greatest scientific precision, and that is the human ear. Aural ability appears to me to be the basis and criterion for a new experience of music that is developing. With this we are in a position to reshape fixed ideas about music into ideas that are more differentiated and flexible.

The author is indebted to **Christian Ginạt** for the following contribution:

Variations in concert pitch are a great problem for anyone with absolute pitch. Such people will find a violin tuned a major second lower extremely difficult to play, for example, because the sense of absolute pitch forces them not only to hear each tone differently in advance, but to inwardly identify it differently as well. What is normally heard as A can not possibly be heard as B.

Compared to this example, the differences in pitch which Maria Renold has worked out are minimal. Listeners with absolute pitch must therefore first of all ask themselves if they actually hear these differences as 'absolute'. Can I, not so far having heard or played a tone that day, really differentiate between a tuning fork at a^1 = 440 Hz and another at a^1 = 432 Hz, i.e., identify the first one I hear correctly? If not, getting used to the new tuning will come with practice and I will become accustomed to it—like any other listener. If I can, then in my experience a certain amount of deliberate flexibility is called for which takes account of the mood of this tuning when listening and playing.

In my view this is not a question of experiencing the individual tones and scales differently but rather of a different 'nuance' into which the listening ear enters with each new tuning, similar to the nuance or 'tone' in which I say a sentence depending on the mood I am in or the effect I wish to produce. One can then train the ability in oneself of evoking the different tunings without the help of the ear, by calling up the differences in nuance inwardly.

I am grateful to Maria Renold for her suggestion that one should pay attention to the very different experiences connected with these 'nuances' and I can only recommend that everybody make use of this possibility and, in spite of and at the same time thanks to the hearing faculty one has had so far, deliberately go deeply into these different qualities.

The contributions by G. Killian and C. Ginat show a way of overcoming the obstacle of fixed absolute pitch, making it into a flexible organ of perception for inherent tone qualities. (Christian Ginat can perform on his viola with equal certainty at a^1 = 440 Hz and at a^1 = 432 Hz, for example.) Working with the c = 128 Hz concert pitch can thus be a liberation for people who have absolute pitch; but not only for them, but for all who wish to open themselves as whole human beings, in music making or listening to the breadth of these tones that are so right for human beings and leave them free.

28 Further Aspects

$a^1 = 432$ Hz—a second concert pitch indication given by Rudolf Steiner

Since the 2nd edition of this book was published, Dieter Marx has drawn the author's attention to a second concert pitch indication by Rudolf Steiner. In the early 1920s Mr and Mrs Ziemann-Molitor told Rudolf Steiner of their intention to build recorders for the children of the newly established Waldorf School and asked his advice. He told them what needed to be taken into account and said: and we can tune them to $a^1 = 432$ Hz! The original Ziemann-Molitor recorders were indeed tuned to this tone, as has been confirmed on the few remaining instruments.

How do the two concert pitches $c = 128$ Hz and $a^1 = 432$ Hz relate to each other? They lie an octave plus a true sixth apart, in other words, a ratio of 27:8. The tone $c = 128$ Hz has shown itself in the course of our deliberations to be the fundamental prime of our age, anchored in the human being, and its true-tone major scale proved to be the Christian ascending inversion of the true Dorian octachord, i.e., the pre-Christian descending Apollonian Sun scale of the Greeks. As the just C major scale it became the centre and model for all major scales in the circle of fifths (see chapters 14–21). The C major scale relates directly to the Christ and to the Mystery of Golgotha. When the human being follows the sequence of the interval degrees in the C major scale[33] from the 'I' experience in the prime to the second 'I' experience in the octave, the existence of God can become an inner experience and a certainty. Human beings can then have both feet on the earth in full awareness of their 'I' and direct their gaze freely to the heavens.

The second concert pitch indication given by Rudolf Steiner was $a^1 = 432$ Hz. This is a true sixth above the octave characterized above, and not just a fifth, of which Rudolf Steiner said that it is the human being. In the sixth, especially the major sixth, the human being opens up to the periphery. The fifth is a firmly imprinted form, the major sixth opens it up and takes us beyond it.

From ancient times—and also by Rudolf Steiner—the tone A was considered to be the Sun tone (see p. 81). The ruler of the Sun is the Archangel Michael, who is the spirit of our age. We may therefore also call $a^1 = 432$ Hz the Michael tone.

In conclusion it may thus be said that when music-making human beings gain an inner certainty of God, i.e., the reality of the spiritual world, in the experience of the interval degrees of the c–c^1 major scale which is connected to the Christ spirit, then Michael opens our eyes again to the spiritual worlds with the tone $a^1 = 432$ Hz, which is part of the twelfth-tone row of C, and the experience of the sixth that belongs to it.

Retuning orchestral instruments

Many musicians have already let themselves be inspired by the first two editions of the book to experiment and listen for the facts that were given in them. Music teachers in Rudolf Steiner schools were particularly impressed by the pleasant effect of the two concert pitches described that they used recorders tuned to $c^1 = 256$ Hz and $a^1 = 432$ Hz. Several of them also had their pianos tuned to the scale of twelve fifths.

The wish also arose among professional musicians to apply the insights in practice. If this intention is to be realized for a full symphony orchestra, all instruments will have to be retuned to $a^1 = 432$ Hz. This is not a problem for the strings as the soundpost rarely needs to be adjusted. It also is not difficult for wind instruments, as shown by the following reports from professional musicians. Names and addresses of instrument builders who deliver instruments tuned at and to the new chamber pitch are given in Appendix 3.

Instructions for retuning an oboe (Ulf Gruendler)

I own a 40-year-old oboe made by Mollenhauer in Fulda. In order to tune to a^1 = 432 Hz I have to pull out (lengthen) the 72 mm reed by about 8.5 mm. To prevent air vortices developing in the space thus created, which would have an unfavourable effect on the onset and end of a tone, I cut an 8.5 mm reed-sleeve from cork. I always leave this in the top of the reed shaft.

It is also possible to gain 1 to 2 Hz in depth by not pushing the head and middle joints of the oboe together completely.

Intonation problems are hardly ever caused by the reed jutting out. Only 'long' tones, i.e., tones with a long wind column—d^1, d^2, e^1 and e^2—are a bit sharp, which is not noticeable in fast passages and in slow passages can be slightly adjusted by means of the embouchure.

The above retuning method applies only to the above-mentioned oboe. I know of an oboist with a Yamaha oboe of a more recent design, which, in spite of the insert, had a tone c^2 that was downright 'sick', while the rest of the intonation was good on this instrument.

Instructions for retuning brass instruments

Horn, trumpet, trombone and tuba (Guenter Blechert)

A brass instrument can usually be taken down in pitch by slightly pulling the crook or tuning slide out (both, of course, for a double horn). Pulling it out too far makes the instrument difficult to play because extra air spaces and vortices result. To avoid this it is necessary to use crooks that are longer by the required amount; these need to be specially made. In keeping with the lower pitch, the valves also need to be pulled out a bit, or possibly lengthened to correspond to the main slide. A good workshop can make these changes without difficulty. Lengthening the mouthpiece has its limitations as this can cause a change in sound quality.

The unusual situation may occur that lengthening the crook gives rise to a 'tone wolf'. This is overcome by increasing the diameter of the particular crook.

The fine intonation needs to be made by the player himself, as designs vary even within the same instrument group.

Instructions for the harp

To play a harp in twelve fifth-tones tuning, the fork disks may need to be replaced with longer ones as the equal-tempered minor seconds and major seconds are smaller than the 9:8 major seconds and their geometric mean tones.

Rudolf Steiner's last important indication relating to music

In August 1924, Rudolf Steiner spoke in Torquay, England about the conscious awareness of initiates. At the end of the last lecture he spoke of how it is possible for human beings to gain an experience of the spiritual world through art in that the artist places something in the sense world 'that rises up to the spiritual world'. Then the following was said:

> The musical element is capable of placing this Christ impulse in music before the world in formed sounds filled with soul and spirit. Ways will be found if music can be inspired by anthroposophical spiritual science. Music will solve the riddle, in a purely artistic way, as to how one can symphonically bring the Christ impulse that lives in cosmos and earth to life in tones.
>
> To do this one only needs to be able to deepen the sphere of the major third by taking the experience of music to a level of mystical intensification. If one experiences this as something that is musically wholly within the human being, and if one then senses the sphere of the major fifth, if one senses the sphere of the fifth to have an enveloping quality, something of a quality where human beings, in growing into the configuration of the fifth reach the limits of what is human and cosmic,

where the cosmic sounds into the human and the human longs to go out to the cosmic, longing stormily to do this, then it is exactly in the realm of music that one can—because of the mystery that takes place between the sphere of the major third and the sphere of the major fifth—experience something of the inner human quality that wants to go out into the cosmic. And if one also manages first to let the life of the cosmos resound in the dissonances of the seventh, where those dissonances speak as the quality which human beings can sentiently experience in the cosmos as they are on their way out into the different regions of the spirit, and if one also succeeds in letting the dissonances float away so that in floating away they assume a distinctive quality, the dissonances in the seventh will receive something in doing so that presents itself to the musical experience like a musical firmament.

And if one then finds, as already hinted at in delicate ways, a minor experience in the major experience, one then finds in this floating away of the dissonances in the seventh, in the way those dissonances come together in a whole, which becomes almost harmonic in its totality, almost consonant because it floats away—if one finds the possibility in this to come out of the dissonances in intensive minor, out of the almost harmonic quality in the floating away of the dissonances, if one finds the way back to the sphere of the minor fifth and from there to the minor third being present in the sphere of the fifth, then one has created the experience, the musical experience, of incarnation, and specifically the incarnation of the Christ.

It will then be possible to find in thus feeling one's way into the sphere of the seventh, which in cosmic sentience is only apparently dissonant, making this into a firmament by having the octave behind it, as it were, but only approximately so, and if one has taken hold of this in developing a feeling for it, and then turns back in the indicated way and finds how the seminal configuration of the third consonances in minor holds the possibility of representing the incarnation as something musical, then, as we go back to the major in this sphere, the 'hallelujah' of the Christ may sound out of this musical configuration, purely as music, purely out of the configuring of the tones. Then the human being will conjure up within the configuration of the tones, in giving this form, something that is immediately supersensible and place it there for sentient musical feeling.*

These words of Rudolf Steiner yield up great mysteries. Maybe the presentations of this book will contribute a little towards coming closer to their solution.

* The passage has been rendered rather literally as the process and transformation gone through in it seems as important if not more so to the editor than the merely informational content. (Translator's note.)

APPENDICES

Appendix 1 How to Build and Use a Monochord

Readers are now given the opportunity to test what has been said in this work for themselves on a three-stringed monochord. Such an instrument is relatively easy to build. For our purposes, a three-stringed is more suitable than a single-stringed monochord because only single tones can be produced on a single string, whereas intervals and even triads are possible with three strings.

A rectangular resonance box is especially useful for series of intervals as this provides a flat surface underneath the strings on which the string lengths for single tones are accurately marked. These markings and a movable bridge enable one to quickly find and fix the correct position for the desired pitch on the string which can then be plucked or bowed. If one shortens one of two unison tuned strings using the movable bridge and then plucks or bows them together, the required interval will be obtained.

Note that all tones and intervals can only sound correctly, if

a) the marking is exact to within 0.5 mm (the string length being only 93 cm);
b) the upper edge of the movable bridge is exactly vertical to the marked division;
c) the strings are tuned in perfect unison.

Table 34 gives the Hz for string-length division with deviations of only 0.5 mm, to show that maximum exactness in the string division is required especially with high tones. So long as the monochord is built with care and skill, it will give good service for the hearing experiments. It has a sonorous tone which carries well, and makes it possible for all tones and intervals discussed in this book to be played. The instrument can also be built to other proportions. However, the exact placing of the tones becomes difficult if not impossible if it is much shorter, and it becomes unmanageable if it is much longer.

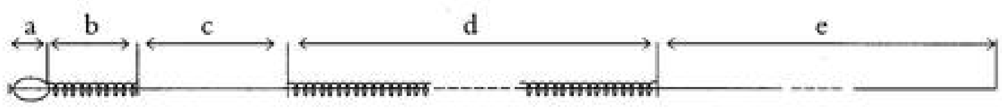
Materials

Resonance box

1 piece of medium hardwood, e.g. mahogany, 1 × 5 × 108 cm, for the base;
2 pieces of top quality three ply 0.5 × 9.5 × 108 cm;
1 piece of top quality five ply 1 × 17 × 108 cm, for the fingerboard;
2 pieces of hardwood each made from 3 layers of wood glued together—two upper pieces together 2.5 cm high, 5 cm wide and 7.5 cm long glued to a lower piece 6 cm high, 5 cm wide and 7.5 cm long—for pins, bridges and screws;
1 piece of wood with 1 cm^2 cross-section and 5 cm long, for the fingerboard;
water-resistant glue;
Araldite Rapid (epoxy resin);
varnish (half gloss).

For the upper construction

From a piano shop:
3 thin, steel strings with a fine copper-wire winding for the tone c = 128 Hz as shown below:



a) eye diameter, 1 cm; b) eye binding, 2.5 cm; c) naked steel-string, core of 1 mm diameter, 4 cm; d) length of string with fine copper-wire winding (making a total diameter of 1.6 mm), 90.5 cm; e) naked steel-string, core 20 cm (installed string length 112 cm, total string length about 120 cm).

three 7 mm-diameter piano pins;
one piano-tuning hammer.

From an ironmongery:
3 10 cm long, thick, wide-headed wood-screws that fit in the eyes of the strings, to hold these;
4 very thin, 3–5 cm long wood-screws to fix the bridges;
2 5 cm long iron angle-profiles 1.5 × 1.5 × 0.2 cm for the bridges;
2 7 cm long brass angle-profiles 1.5 × 1.5 × 0.2 cm for the movable bridges;
2 sheets of 7 × 14 × 0.2 cm Plexiglas as bases for the movable bridges;
1 sheet of Plexiglas 17 × 19 × 0.2 cm to cover the fingerboard;
6 5-mm brass screws to attach the Plexiglas;
1 piece of white cardboard for the fingerboard 17 × 93 × 0.2 cm;
1 200-cm metal ruler with mm divisions.

1 tuning fork at c = 128 or 256 Hz, according to the indications on p. 77.

Assembly

The three pieces of wood for each of the two hardwood pieces need to be glued together with water-resistant glue so that the grain of one block is at 60° to the grain of the other. The two upper pieces should have a combined thickness of 2.5 cm, the lower one 6 cm. It is best to let a professional make these hardwood pieces and the boards, iron and brass bridges. Accurately made, this will save a lot of time and frustration. After this preparatory work one can begin with the assembly itself.

First glue the pieces of hardwood to be on and parallel to the base with waterproof glue, one at each end, perfectly vertical and exactly 93 cm apart and held in place with clamps until dry.

Place the side-boards together, with the outsides facing each other. Draw a line on the inside face of the boards 4.5 cm away from the upper edge and find the middle of that line. From there mark the centre of the sound holes, which should be 6.5, 19.5 and 32.5 cm from both sides of the middle. Drill the sound holes slowly to avoid splinters, using a 2.5 cm drill bit. Experience has shown that this number and size of sound holes give the best results for this size monochord. Now glue these one at a time, outer faces outwards, to the sides. The glue must be spread evenly to cover the whole surface of the hardwood blocks and the sides of the base. Clamp together. Mark the exact middle of the sides of the hardwood blocks on the upper edge of the side pieces. Glue the 1 × 1 × 5 cm piece of wood to the middle of the inner upper edges of the sides. The piece prevents the sides from bending inwards and supports the fingerboard and must therefore be set in place very exactly.

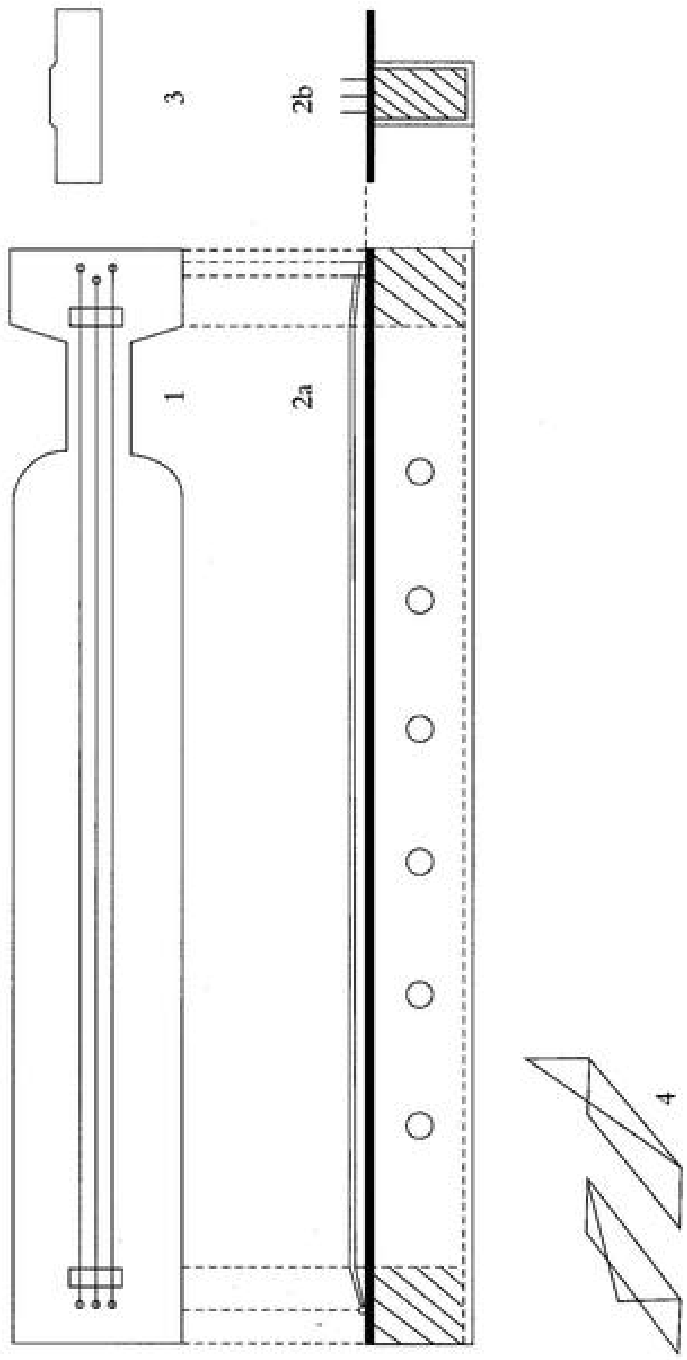
When the lower part of the monochord is finished, draw the middle of the string lengths on the underside and edges of the fingerboard. Then mark the places where the iron bridges will be. This is 7.5 cm from the ends and therefore directly above the edge of the hardwood blocks. The lines, exactly 5 cm long, must be exactly in the middle of the longitudinal axis and exactly 93 cm apart, at an exact right angle to the longitudinal axis and parallel to the end edges of the resonance box.

To bow the strings of the monochord, the fingerboard, cardboard and Plexiglas need to be reduced to the width of the resonance box on both sides and at the same end as the tuning pins. This is 11 cm long and begins at the point next to where the right-hand iron bridge will be. The cut is made rounded on the left (so that the bow cannot get caught) and has a 60° angle on the right (see Fig. 5, 1).

Now glue and clamp the fingerboard to the resonance box so that its middle is exactly above the middle of the latter. The fingerboard will then overlap the resonance box on either side by nearly 4 cm and be the same length at the ends.

The two iron bridges now need to be made from the 5 cm long iron angle-profile pieces. Two holes big enough for the two 3 cm long thin wood-screws are drilled into the foot of the bridge. Roughen

Fig. 5
The construction of a monochord



1. The fingerboard from above with the pins (right), screws, strings and bridges
2. The resonance box and fingerboard: a) longitudinal section with resonance holes,
   b) narrow section
3. The bridge (shown proportionally larger)
4. Two movable bridges. Plexiglas base, brass angle-profile

up the underside of the foot with a file so that the Araldite will adhere better. To allow all three strings to be played with a cello bow, divide the top of the bridge into three equal lengths (one for each string) and file the two outer lengths down to be 3 mm lower than the middle length (see Fig. 5, 3). Sand the edges of the bridge down before it is mounted, so that the string will not be damaged. Once again it is advisable to mount one bridge at a time. The position of the second bridge can then be measured from the one already mounted so that they are parallel to the ends and exactly 93 cm apart. When dry, support the bridges further with a block, screwing it on so that the screw heads are counter-sunk into the wood. This will prevent the bridges from moving and getting out of line which would make accurate playing of the monochord impossible.

Varnish the wooden parts with a clear, light furniture varnish. NB: only the part of the fingerboard that is not covered by the cardboard is varnished. Two or three coats is enough. Make sure that the directions for use are followed and that the varnish is given enough time to dry between coats.

Now set the pins at the right-hand end. The holes for the three pins need to be drilled displaced from one another so that it will be easy to place the tuning hammer on the pins. As the thread is left-handed, they must be at an angle so that the pins lean towards the right, away from the strings. The drill-bit needs to be about 0.5 mm less in diameter and the holes 3–5 mm longer than the thread of the pins. This air space under the pins helps to make them easily movable. The pins need to be screwed and not hammered in. Hammering in may break the wood-fibres, which are necessary to help keep the pin in place when the string has been tuned. Take care not to damage or round off the edges of the pins, otherwise they cannot be turned any more and the strings would not be tuneable.

On the left-hand side, drill the holes for the screws that will hold the eyes of the strings with a drill-bit that is thinner and to a depth shorter than the screws (the exact size and depth will depend on the type of wood and should therefore be discussed with a carpenter), so that they lie in the middle of the eye and are angled away from the strings. The screws are then put through the eye of the strings and screwed in.

The last 4 mm of each string is bent over at a right angle and passed through the small hole in the rest-pin opposite. Then, using the tuning hammer, the string is carefully and evenly wound onto the pin without cross-overs, proceeding downwards from the hole until the string is stretched evenly over both bridges.

Tuning the monochord using a tuning fork

Each string is tuned exactly to c = 128 Hz using the tuning fork. The tuning fork has a handle, by which it is held, and two prongs. To make it sound, strike one of the prongs against a wooden object or your kneecap. Then place the foot of the tuning fork on the resonance box of the monochord or on the table-top. This amplifies the tone. Turn the rest-pin with the tuning hammer until the string sounds at the same pitch as the tuning fork. If the tuning fork is 256 Hz the harmonic in the middle of the string needs to sound in tune with it. The place of this harmonic is in the exact centre between and parallel to the two bridges. The point must be exactly marked so that the fingerboard may be exactly divided and the monochord used for the hearing experiments described in this book.

To play the tones, two movable bridges need to be made from brass and Plexiglas (see Fig. 5, 4). The 1.5 × 1.5 × 0.2 cm brass angle-profiles are glued, not yet cut at an angle, one along the left, the other along the right-hand edge of the 3 × 14 × 0.2 cm Plexiglas bases with Araldite (roughen the surfaces to be glued together so that the glue will hold). When the glue has dried, angle the top of the bridge and smooth the edges in such a way that the string is stopped by the middle of the bridge.

Marking the fingerboard*

Mark the measurements on the sturdy piece of white cardboard. Fix the cardboard firmly to the table or drawing board. First draw twelve parallel lines, one for each of the scales discussed in the book, but so that they are not immediately below any of the strings. Four lines should be in front of the first string, the other eight in between and behind the other strings. To help the eye find the different tones and scales it is advisable to draw the lines in different colours. On our monochord the first two are red, the third is green, the fourth to tenth are blue, the eleventh and twelfth black. Extra lines in other colours again can be added if yet other scales and tones are to be investigated. It has proved practical to write the names of the scales on the left-hand side of the fingerboard.

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* Refer to Tables 34 and 35.

Now the divisions can be marked. These are easily calculated if the frequencies of the fundamental of the string and the required tone are known. For our purposes it is best to tune all three strings to the fundamental of c = 128 Hz. If the required tone is higher than the string fundamental then string length L′ with the frequency F′ can be worked out for a string of length L and frequency F by using the following formula:

$$\underline{L \times F} = L'$$

E.g. The harmonic for the octave and double octaves of a string 93 cm in length with a fundamental of c = 128 Hz can be worked out as follows:

$$\frac{93 \text{ cm} \times 128 \text{ Hz}}{256 \text{ Hz}} = 46.5 \text{ cm (= string middle)}$$

$$\frac{93 \text{ cm} \times 128 \text{ Hz}}{512 \text{ Hz}} = 23.25 \text{ cm (= string quarter)}$$

For the modes, the length of string L′ is important. As the positions are equidistant, use the following formula, with N the numerator and D the denominator:

$$L' = \frac{L \times N}{D}$$

These divisions must be very accurately marked if the monochord is to serve for hearing observations. This becomes evident when the divisions of the string for the altered fifth tones are compared with those for the just tones. The difference at the left-hand end of the monochord is less than 1 mm. If one wants to be able to play and hear the tones correctly, the margin of error has to be less than 0.25 mm.

On our monochord the scales are marked in the same order as for Tables 34 and 35. These are given in order to save the reader the lengthy task of calculating all the string lengths. Scales and tones that are not given here can easily be calculated using the above formulae. The division of the string length is measured from the bridge at the left-hand end and marked with a fine coloured ballpoint pen at right angles to the lines, with the name of the relevant tone above the line and the Hz below.

When marking is finished you will find that some tones appear in several scales. For example:

just d♭ = 136.533 Hz = 30/32 of the Saturn mode;
true d = 144.000 Hz = 2nd degree of the scale of twelve fifths and 32/26 (mese) of the Jupiter mode;
just e♭ = 153.600 Hz = 30/36 of the Jupiter mode and 40/48 of the Venus mode and 3rd degree of the diatonic just C minor scale;
just e = 160.000 Hz = 32/40 (mese) of the Mars mode and 3rd degree of the just C major scale;
true f = 170.667 Hz = 4th degree of the scale of twelve fifths and the just C major scale, 24/32 of the Saturn, 30/40 of the Mars and 36/48 of the Venus mode;
true g = 192.000 Hz = fifth degree of the scale of twelve fifths and the just C major scale, 1st degree of the just G major and diatonic just G minor scales, 24/36 of the Jupiter and 32/48 of the Venus mode;
just a♭ = 204.800 Hz = sixth degree of the diatonic just C minor scale and 20/32 of the Saturn and 30/48 of the Venus mode.

It is helpful in work with the monochord to remember such tones.

Now place the cardboard on the fingerboard. The middle (lengthwise) of the cardboard (46.5 cm) needs to be exactly above and parallel to the middle of the fingerboard. Shorten the cardboard at both ends by 1 mm so that it lies flat. The total length will thus be 92.8 cm. The cardboard needs to be held with paper clips so that it can be cut to the form of the fingerboard on the right-hand side. Before gluing with Araldite check that the midline is the perfect octave of c = 128 Hz by playing it and checking it against the tuning fork. Adjust if necessary, then glue with the utmost care so that it does not move. If it moves, the whole cardboard piece has to be torn out, a new one drawn and all the work redone.

When the glue has dried attach the already cut Plexiglas cover to the fingerboard with small brass screws. This will protect the cardboard from wear and tear and make it easier to move the movable bridges.

Playing the monochord

The monochord is now finished and can be played. It can be placed on a table or, which is more practical, attachments can be fixed to the bottom of the monochord (under the hardwood blocks) and table legs screwed into these. These are easy to obtain and make a free-standing instrument which is easy to dismantle for transportation or storage in a bag made from impregnated sailcloth. The strings can be plucked or bowed with a cello bow.

To play intervals from c = 128 Hz, a single movable bridge is sufficient. This is placed under and at right angles to one of the outer strings so that it lightly touches it, though without raising it as this would affect the intonation. The upper edge of the bridge must be exactly above the line on the fingerboard. The string is held on to the bridge by a finger of the left hand and the two strings are plucked or bowed together or one after the other by the right hand. Thus the difference between the dissonant true intervals and the imperfect just consonants is easily made audible, for example. Two movable bridges will be needed to play the intervals that lie between two marks. This is done as described for a single bridge above, only now two strings need to be held with the left hand instead of one, e.g. place one of the bridges exactly on the line for fau = 176 Hz = 32/44 = mese of the pre-Christian Greek Sun mode (seventh line) and the other on a = 220 Hz, the first lower octave of a^1 = 440 = the concert pitch with equal-tempered tuning (11th line). This gives the just major third that arises between the two tones. All scales described in this book can be played based on c = 128 Hz as common prime. Do not forget to tune all three strings in exact unison.

The author now wishes you much pleasure in your own investigations!

Table 34

Measurements for the string divisions of the monochord, scale of twelve fifths, all altered fifths, all just tones and aulos modes on the generic tone c^5 = 4096 Hz for a monochord with the string-length of 93 cm on the fundamental c = 128 Hz

Usual division: genuineness and falseness of the intervals is not evident from the numbers.													
First line, red, the twelve fifth-tones scale. Lower octave:													
Tone:	t. c	delis	t. d	elis	t. e	t.f	gelis	t. g	alis	t. a	belis	t. b	t. c^1
Hz:	128	135.765	144	152.735	162	170.667	181.019	192	203.645	216	229.103	243	256
cm:	93	87.68	82.67	77.94	73.48	69.75	65.76	62	58.45	55.01	51.96	48.99	46.5
Higher octave:													
Tone:		delis^1	t. d^1	elis^1	t. e^1	t. f^1	gelis^1	t. g^1	alis^1	t. a^1	belis^1	t. b^1	t. c^2
Hz:		271.53	288	305.47	324	314.333	362.038	384	407.29	432	458.206	486	512
cm:		43.84	41.34	38.97	36.74	34.88	32.88	31	29.22	27.5	25.98	24.49	23.25
Second line, red, all altered t.-tones (Hz: can be found in Table 19). Lower octave:													
tone	cm	tone	cm	tone	cm	tone	cm	tone	cm	tone	cm	tone	cm
t. c	93	t. b♯♯	85.91	t. c♭	78.37	t. g♭♭	70.69	t. e♯♯	64.44	t. g♯	58.06	t. b♭	52.31
t. b	91.75	t. c♭♭	83.8	t. d♯	77.41	t. e♯	68.81	t. a♭♭	62.85	t. b♭♭	55.87	t. a♯	51.61
t. d♭	88.28	t. c♯♯	81.55	t. f♭	74.48	t. g♭	66.21	t. f♯♯	61.17	t. g♯♯	54.37	t. c♭	49.66
t. c♯	87.09	t. f♭♭	79.54	t. d♯♯	72.49	t. f♯	65.32	t. a♭	58.85	t. c♭♭	53.03	t. a♯♯	48.33
												t. d♭♭	47.13
Higher octave:													
t. b♯^{1}	45.88	t. e♭♭^{1}	41.9	t. d♯^{1}	38.7	t. e♯^{1}	34.4	t. a♭♭^{1}	31.43	t. b♭♭^{1}	27.94	t. a♯^{1}	25.8
t. d♭^{1}	44.14	t. c♯♯^{1}	40.78	t. f♭^{1}	37.24	t. g♭^{1}	33.1	t. f♯♯^{1}	30.58	t. g♯♯^{1}	27.18	t. c♭^{1}	24.86
t. c♯^{1}	43.54	t. f♭♭^{1}	39.77	t. d♯♯^{1}	36.24	t. f♯^{1}	32.66	t. a♭^{1}	29.42	t. c♭♭^{1}	26.52	t. a♯♯^{1}	22.16
t. b♯♯^{1}	42.95	t.e♭^{1}	39.18	t. g♭♭^{1}	35.34	t. e♯♯^{1}	32.22	t. g♯^{1}	29.03	t. b♭^{1}	26.16	t. d♭♭^{1}	23.5
Third line, green, all j. tones (Hz: can be found in Table 19). Lower octave:													
tone	cm	tone	cm	tone	cm	tone	cm	tone	cm	tone	cm	tone	cm
j. d♭♭	93.1	l. j. d.	83.7	h. j. e♭	77.5	j. g♭♭	69.83	l. j. g	62.77	l. j. a	55.8	h. j. b♭	51.64
j. b♯	92.9	j. e♭♭	82.76	l. j. e	74.4	j. e♯	69.67	j.a♭♭	62.07	j.b♭♭	55.17	l. j. b	49.6
j. c	91.85	j. c♯♯	82.57	l. j. f♭	73.56	h. j. f	68.89	j. f♯♯	61.93	j. g♯♯	55.05	j. c♭	49.04
l. j. c♯	88.15	h. j. d	81.65	j. d♯♯	73,40	l. j. f♯	66.13	h. j. g	61.23	l. j. a	54.43	j. a♯♯	48.93
j. d♭	87.19	l. j. e♭	79.45	h. j. e	72.57	j. g♭	65.39	j. g♯	58.78	l. j. b♭	52.97	h. j. b	48.38
h. j. c♯	86.01	j. d♯	78.4	l. j. f	70.62	h. j. f♯	64.44	j. a♭	58.12	j. a♯	52.25	j. c	47.08

(*Continued*)

Table 34 (cont.)

Higher octave:

j. d♭♭^{1}	46.55	l. j. d^1	41.85	h. j. e♭^{1}	38.75	j. g♭♭^{1}	34.92	l. j. g^1	31.38	l. j. a^1	27.9	h. j. b♭^{1}	25.83
j. b♯^{1}	46.45	j. e♭♭^{1}	41.38	l. j. e^1	37.2	j. e♯^{1}	34.84	j. a♭♭^{1}	31.04	j. b♭♭^{1}	27.58	l. j. b^1	24.8
j. c^1	45.92	j. c♯♯^{1}	41.28	l. j. f♭^{1}	36.78	h. j. f^1	34.44	j. f♯♯^{1}	30.96	j. g♯♯^{1}	27.52	j. c♭^{1}	24.52
l. j. c♯^{1}	44.08	h. j. d^1	40.82	j. d♯♯^{1}	36.7	l. j. f♯^{1}	33.08	h. j. g^1	30.62	l. j. a^1	27.22	j. a♯♯^{1}	24.46
j. d♭^{1}	43.59	l. j. e♭^{1}	39.72	h. j. e^1	36.28	j. g♭^{1}	32.7	j. g♯^{1}	29.39	l. j. b^1	26.48	h. j. b^1	24.19
h. j. c♯^{1}	43	j. d♯^{1}	39.2	l. j. f^1	35.31	h. j. f♯^{1}	32.22	j. a♭^{1}	29.06	j. a♯^{1}	26.12	j. c^1	23.54

Fourth line, blue, tones of the seven aulos modes derived from the common generic tone c^5 = 4096 Hz: Division 32/32, Hypodorian: on these tones, all the seven aulos modes sound as individual scales and leave the consciousness of the modern Western human being totally free. (See also Tables 5 and 15.)

Lower octave:

Tone:	t. c	j. d♭	m. d	m. e	t. f	m. geo	j. a♭	t.b♭	t. c^1
Degree:	32/32 (mese)	30/32	28/32	26/32	24/32	22/32	20/32	18/32	16/32 (mese)
Hz:	128	136.533	146.286	157.538	170.667	186.182	204.800	227.555	256
Planet:	Saturn	Moon	Moon	Mercury	Venus	Sun	Mars	Jupiter	Saturn
cm:	93	87.19	81.37	75.56	69.75	63.94	58.12	52.31	46.5

Higher octave:

Tone:		j. d♭^{1}	m. d^1	m. e^1	t. f^1	m. geo^1	j. a♭^{1}	t. b♭^{1}	t. c^2
Degree:		15/32	14/32	13/32	12/32	11/32	10/32	9/32	8/32 (mese)
Hz:		273.067	292.571	315.077	341.333	372.364	409.6	455.111	512
Planet:		Moon	Moon	Mercury	Venus	Sun	Mars	Jupiter	Saturn
cm:		43.59	40.69	37.78	34.87	31.97	29.06	26.15	23.25

Still higher octaves:

Tone:	t. c^3	t. c^4	t. c^5
Degree:	4/32 (mese)	2/32 (mese)	1/32 (generic tone)
Hz:	1024	2048	4096
Planet:	Saturn	Saturn	Saturn
cm:	11.62	5.81	2.91

Abbreviations: m. = modal; j. = just; t. = true; h. = high; l. = low

Table 35

The measurements for the lengths of the six other pre-Christian Greek aulos modes, the equal-tempered scale at a^1 = 440 Hz and at c = 128 Hz

Usual division:

Fifth line, blue, pre-Christian Greek Jupiter aulos mode. Generic tone t. d^5= 4608 Hz: Hypophrygian 36/36: Lowest octave

Tone:	t. c	t. d	j. e flat		t. g		j. b flat	t. c^1	
Degree:	36/36	32/36 (mese)	30/36	26/36	24/36	22/36	18/36		
Hz:	128	144	153.6	177.231	192	209.454	230.4	256	
cm:	93	82.67	77.5	67.17	62	56.83	51.67	46.5	

Higher Octaves

Tone:	t. d^1	t. d^2	t. d^3	t. d^4	t. d^5
Degree:	16/36 (mese)	8/36 (mese)	4/36 (mese)	2/36 (mese)	1/36 = g.t.
Hz:	288	576	1152	2304	4608
cm:	41.34	20.67	10,34	5.17	2.58

Sixth line, blue, pre-Christian Greek Mars aulos mode. Generic tone j. e^5= 5120 Hz: Hypolydian 40/40: (lower tetrachord with 30/40 = the origin of our modern j. c ma. scale.) Lowest octave:

Tone:	t. c	j. d	j. e	t. f			j. a		t. c^1
Degree:	40/40	36/40	32/40 (mese)	30/40	or 28/40	26/40	24/40	22/40	20/40
Hz:	128	142.222	160	170.667	or 182.857	196.923	213.333	232.727	256
cm:	93	83.7	74.4	69.75	or 65.10	60.45	55.8	51.15	46.5

Higher octaves:

Tone:	j. e^1	j. e^2	j. e^3	j. e^4	j. e^5
Degree:	16/40 (mese)	8/40 (mese)	4/40 (mese)	2/40 (mese)	1/40 = g.t.
Hz:	320	640	1280	2560	5120
cm:	37.2	18.6	9.3	4.65	2.32

Seventh line, blue, pre-Christian Greek Sun aulos mode. Generic tone m. fau^5= 5632 Hz: Dorian 44/44. (Schlesinger tuning. This tuning can have a slight somnambulous effect.) Lowest octave:

Tone:	t. c			m. fau					t. c
Degree:	44/44	40/44	36/44	32/44 (mese)	30/44	or 28/44	26/44	24/44	22/44
Hz::	128	140.8	156.444	176	187.733	or 201.143	216.615	243.667	256

(Continued)

Table 35 (cont.)

Planet:	Sun	Mars	Jupiter	Saturn	Moon	Moon	Mercury	Venus	Sun
cm:	93	84.54	76.09	67.64	63.41	or 59.18	54.95	48.85	46.5

Higher octaves:

Tone:	m. fau^1	m. fau^2	m. fau^3	m. fau^4	m. fau^5
Degree:	16/44 (mese)	8/44 (mese)	4/44 (mese)	2/44 (mese)	1/44 = g.t.
Hz:	352	704	1408	2816	5632
Planet:	Saturn	Saturn	Saturn	Saturn	Saturn
cm:	33.82	16.91	8.45	4.23	2.11

Eighth line, blue, pre-Christian Greek Venus aulos mode. Generic tone t. g^5= 6144 Hz: Phrygian 48/48. (with 30/48 = the origin of the harmonic mi. scale, with 28/48 = origin of the melodic minor scale.) Lowest octave:

Tone:	t. c		j. e flat	t. f	t. g	j. a flat			t. c^1
Degree:	48/48	44/48	40/48	36/48	32/48 (mese)	30/48	or 28/48	26/48	24/48
Hz:	128	139.636	153.6	170.667	192	204.8	or 219.429	236.308	256
cm:	93	85.25	77.5	69.75	62	58.12	or 54.25	50.37	46.5

Higher octaves:

Tone:	t. g^1	t. g^2	t. g^3	t. g^4	t. g^5
Degree:	16/48 (mese)	8/48 (mese)	4/48 (mese)	2/48 (mese)	1/48 = g.t.
Hz:	384	768	1536	3072	6144
cm:	31	15.5	7.75	3.87	1.94

Ninth line, blue, pre-Christian Greek Mercury aulos mode. Generic tone m. a flat^5= 6656 Hz: Lydian 52/52. Lowest octave:

Tone:	t. c					m. a flat			t. c^1
Degree:	52/52	48/52	44/52	40/52	36/52	32/52 (mese)	30/52	or 28/52	26/52
Hz:	128	138.667	151.273	166.4	184.888	208	221.867	or 237.714	256
cm:	93	85.85	78.69	71.54	64.39	57.23	53.66	or 50.08	46.5

Higher Octaves:

Tone:	m. a flat^1	m. a flat^2	m. a flat^3	m. a flat^4	m. a flat^5
Degree:	16/52 (mese)	8/52 (mese)	4/52 (mese)	2/52 (mese)	1/52 = g.t.
Hz:	416	832	1664	3328	6656
cm:	28.62	14.31	7.15	3.57	1.78

Tenth line, blue, pre-Christian Greek Moon aulos mode. Generic tone low j. b flat^5= 7168 Hz. Myxolydian 56/56: (This mode can cause women to have heavy periods.)

Lowest octave:

Tone:	t. c						low j. B flat	t. c^1
Degree:	56/56	52/56	48/56	44/56	40/56	36/56	32/56 (mese)	28/56
Hz:	128	137.846	149.333	162.909	179.2	199.111	224	256
cm:	93	86.36	79.71	73.07	66.43	59.78	53.14	46.5

Higher octaves:

Tone:	low j. B flat^1	low j. B flat^2	low j. B flat^3	low j. B flat^4	low j. B flat^5
Degree:	16/56 (mese)	8/56 (mese)	4/56 (mese)	2/56 (mese)	1/56 = g.t.
Hz:	448	896	1792	3584	7168
cm:	26.57	13.28	6.64	3.32	1.66

Eleventh line, black, the tones of the equal-tempered scale at the concert pitch a^1= 440 Hz: Octave in the middle of the monochord:

tone:	e.t. a	e.t. B flat	e.t. b	e.t. c^1	e.t. D flat^1	e.t. d^1	e.t. E flat^1	e.t. e^1	e.t. f^1	e.t. f sharp	e.t. g^1	e.t. A flat	e.t. a^1
Hz:	220	233.1	246.9	261.7	277.2	293.7	311.1	329.6	349.2	370	392	415.3	440
cm:	54.11	51.07	48.21	45.49	42.94	40.53	38.26	36.11	34.09	32.17	30.37	28.67	27.05

Twelfth line, black, the tones of the e.t. scale at the concert pitch c = 128 Hz: Lower octave:

Tone:	t. c	d flat	d	e flat	e	f	f sharp (gelis)	g	a flat	a	b flat	b	t. c^1
Hz:	128	135.6	143.7	152.2	161.3	170.9	181.018	191.8	203.2	215.3	228.1	241.7	256
cm:	93	87.79	82.84	78.21	73.8	69.65	65.76	62.06	58.58	55.29	52.18	49.25	46.5

Abbreviations: m. = modal; j. = just; t. = true; h. = high; l. = low; g.t. = generic tone

Appendix 2 Graphic Representations of the Scale of Twelve Fifths

by Paul Davis

It is probably necessary to say that I did not have the benefit of having known of this book while doing the work that follows. Although this circumstance caused me to explore many unfruitful alleys I did have the advantage of having no opportunity for prejudice regarding the author's methods!

Maria's tuning was demonstrated to me courtesy of Bevis Stevens during a visit to the Goetheanum in 1999. As I played scales and raw intervals I was sceptical. I could hear differences from what I was accustomed to on a piano. These were not terrible but I could not help asking—why make it different? It was when I began playing actual music that the answer came. My prior experiences with alternate temperaments have been largely ones of wincing to various degrees. I accepted intellectually that equal temper was the 'least bad'. An exception to this was my sojourn with the viola d'amore where one tunes to a key and stays in it (or its relations). The effect I found almost too consonant. The played and the sympathetic strings would harmonically relate to each other so well that one would become lost in the sympathy of them all. Any played note could only be heard over the wash of matching overtones by being just a bit louder for a moment. In a short time one would simply hear a continuous key chord. I have also found this to be a quality of the Pythagorean tuning. The tones are so 'social' that hardly any individuality is possible without stepping on dreadful wolves.

The customary equal temperament on the other hand allows only the octave to be 'perfect' or consonant. No tones actually quite match the overtones of others. Here a stepping forward of each note occurs; very individual—even 'antisocial'.* The effect that struck me with Maria's tuning was that of a balance between the social and anti-social. The tones were assertive and individual, and yet somehow consonant and related in a certain but subtle way. Returning home I recalled this experience on many occasions. As luck would have it I found that a friend of mine had attended a class on the twelve fifths tones tuning some years before. He had three tuning forks and a few (incomplete) sheets on the subject which he loaned to me. So began my journey to attempt to reproduce the tuning at home and to penetrate the principle behind the effect. As it turns out all but one of the sheets referred only to an earlier exposition based on perfect fifths and octaves (referred to in Graphs 1–4 as Renold 1). This was fairly easy to understand and to tune—but the effect was not that which I remembered and sought. The odd sheet was produced by Lothar Thomma and was there completely out of context. It was essentially like tearing Table 31 out of this book, handing it over, and saying, 'Now, what do you make of this?' I used the given pitches and the beating interaction of overtones to calculate frequencies for each note. I derived three different octave widths, all sorts of apparently chaotic interval widths and knew not how to proceed. After many false starts in attempts to find an underlying principle through analysis of this sheet I finally gave up and just played with tuning the piano.

I am neither a real piano tuner nor a pianist but listen (with a string player's ear) to both difference tones and beats with ease. It is generally the case that piano tuners develop a keen sense for beat count and annihilate their perception of difference tones.

I found, acoustically, a tuning that produced an effect that matched to my satisfaction what I remembered. Of course this would differ in the particulars from that at the Goetheanum because I was working with a different piano. Next I purchased a precision tuner and used it to document my own

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* For an even more antisocial sound, it is interesting to hear the equal-tempered 19th root of three-semitone tuning based on perfect 3:1 19ths or second harmonics. Although nothing is offensive in the elements (scale, triads etc.), the musical effect is one of utter sterility and loneliness.

results (to +/− .05 cents). Not surprisingly the results were even more chaotic and no principle was revealed through analysis.

On a long holiday on a Greek island (in winter!) I found myself with no piano, but a computer and lots of time. I decided to abandon the analytic approach and try rather the synthetic. I attempted to re-create mathematically what I had experienced with my ear while tuning. Here follows an explanation of my results. The examples are crude for clarity.

The perfect 2:1 octave produces a difference tone identical to and indistinguishable from the lower tone of the octave.

[400 − 200 = *200*]

A widened octave produces a difference tone of a narrowed octave (as well as a beat).

[401 − 200 = *201*; 401:200 is 2.005:1; 401:*201* is *1.995:1* (*201* − 200 = 1)].

That difference tone, not readily apparent, I will call the *complement* to the lower tone.

Taking the higher octave tone, now tune the fifth above (Also the second harmonic to the lower tone). How shall this be tuned? A perfect fifth will produce a difference tone of a perfect octave below the lower tone of the fifth.

[600 − 400 = *200*]

If one widens the fifth the corresponding difference tone becomes higher. That is, the relation of the lower note to the difference octave becomes narrow.

[601 − 400 = *201*; 601:400 is 1.503:1; 400:*201* is *1.990:1*].

Let's tune the fifth so that that difference tone is narrow by the same degree as the wide-octave complement tone.

It follows also with the fourth. The perfect fourth produces a difference tone two perfect octaves below the upper tone. A wide fourth will make a higher difference tone. Match this to the octave's complement tone.

The above is a verbal description of what I believe may account for the twofold properties of individuality and association between the tones. There appears at first to be a departure of harmonic relationships when the simple intervals are widened, but a reassociation is found by making consonant the complementary difference tone. This may very well account for the mysterious taming of the expected annoying beats. If the complement tones are substantially out of phase, the beats may die of destructive interference.

Taking the above and moving into the ideal world of mathematics I calculated, from one given pitch, very nearly the table produced by Mr Thomma. The formulae used were as follows:

L = Lower pitch of an interval to be raised
H = Higher pitch of an interval to be lowered
R = Octave ratio (the variable in search of a solution)

To move up a fifth	2L − L/R
To move down a fifth	HR/(2R − 1)
To move up a fourth	2L/(1 + (2/R ^ 2))
To move down a fourth	H/2 + H/R ^ 2
To find the geometric mean (for gelis from C)	LR ^ (1/2)

Table 36 shows the results along with their difference from those derived from Table 31.

Of course there are several ways of judging the closeness of fit. I chose the result which had the least sum of all deviations in the most sensitive realm: that of the beat count. I found a very good fit here with an octave of 1202.8 cents. From another means of judging nearness of fit, that of the least range of deviation through the compared elements, a good fit was found at 1203.234 cents. It is the former that are represented by the graphs here. I have consequently discovered that the actual intended octave is to be 1203.35 cents. That is astonishingly close to the latter 'fit'.

Table 36*

Synthesize Renold 2 using a consonant complementary difference tone and show deviation from Lothar Thomma's tabulation (Table 33) P. B. Davis 22.04.2002

Octave width in cents: 1202.800
makes an octave ratio of 2.0032373

interval	beats	dev.	cents	dev.
CF P4	1.10	−0.00	499.910	0.0
FA M3	19.49	−0.01	405.955	0.0
CG P5	0.41	−0.25	702.887	−0.6
DA P5	0.47	−0.28	702.887	−0.6
Dg M3	10.10	−0.71	398.422	−0.9
DG P4	1.24	−0.75	499.910	−1.1
EA P4	1.40	0.00	499.910	0.0
CE M3	14.60	0.00	405.955	0.0
BE P4	1.05	0.02	499.910	0.0
Bg F5	−2.77	−0.11	695.354	−0.3
EB P5	0.52	−0.33	702.887	−0.6
gB F4	7.89	−0.25	507.443	−0.3
dg P4	1.17	0.00	499.910	0.0
dF FM3	9.52	0.22	398.419	0.3
da P5	0.44	0.26	702.887	0.5
Ea FM3	11.36	0.27	398.422	0.3
ea P4	1.32	−0.01	499.910	0.0
Be FM3	8.51	0.23	398.422	0.3
eG FM3	10.71	−0.76	398.419	−0.9
be P4	0.99	0.04	499.910	0.1
bF F5	−2.61	−0.09	695.351	−0.2
bD FM3	8.02	0.22	398.419	0.3
eb P5	0.49	−0.32	702.887	−0.6
Fb F4	7.44	−0.24	507.446	−0.3
bb P8	0.74	−0.14	1202.797	−0.5
BB P8	0.78	−0.16	1202.797	−0.6

Constants and Definitions

C4 = 256 cps

L = lower, H = higher, R = octave ratio

Up 4th	$2L/(1 + (2/R'^2))$	
Down 4th	$H/2 + H/R'^2$	
Up 5th	$2L - L/R$	
Down 5th	$HR/(2R - 1)$	
Up tt	$L\sqrt{R}$	(used only for F♯4)

Tuning sequence: C4 F4 C5
C4 G4 D4 A4 E4 B4 B3
g4 d4 a4 e4 b3 b4

note (from)	Hz	dev.
b3 (e4 ↓4)	228.67	0.03
B3 (E4 ↓4)	242.48	0.00
C4 (k)	256.00	0.00
D4 (g4 ↓4)	271.46	−0.04
D4 (G4 ↓4)	287.84	0.09
e4 (a4 ↓4)	305.22	0.05
E4 (A4 ↓4)	323.65	0.00
F4 (C4 ↑4)	341.70	0.00
g4 geo. mean	362.33	−0.06
G4 (C4 ↑5)	384.21	−0.12
a4 (d4 ↑5)	407.40	0.06
A4 (D4 ↑5)	432.00	0.00
b4 (e4 ↑5)	458.08	−0.08
B4 (E4 ↑5)	485.74	−0.16
C5 (F4 ↑5)	512.83	−0.16

MaxΔ–MinΔ	1.029		1.676		0.257
Σ|Δ|	5.667		9.235		0.714

Octave values giving best fits according to *narrowest range* (MaxΔ–MinΔ) and *smallest sum* (**Σ|Δ|**) of deviation:

MaxΔ–MinΔ:	1202.786	1.029	1203.234	1.531	1202.570	0.248
Σ|Δ|:	1202.800	5.667	1202.815	9.200	1202.800	0.714

Consequent Hz for whole piano range

	C	delis	D	elis	E	F	gelis	G	alis	A	belis	B
0	A=432.00									26.826	28.446	30.163
1	31.845	33.768	35.807	37.968	40.261	42.506	45.072	47.794	50.679	53.739	56.983	60.424
2	63.794	67.645	71.729	76.060	80.652	85.150	90.291	95.742	101.522	107.652	114.151	121.043
3	127.793	135.509	143.690	152.366	161.564	170.575	180.873	191.793	203.373	215.651	228.671	242.477
4	256.000	271.456	287.845	305.224	323.651	341.701	362.332	384.207	407.404	432.000	458.082	485.738
5	512.828	543.790	576.621	611.435	648.349	684.507	725.835	769.656	816.125	865.397	917.646	973.047
6	1027.314	1089.339	1155.106	1224.846	1298.795	1371.228	1454.017	1541.801	1634.888	1733.593	1838.259	1949.242
7	2057.950	2182.200	2313.948	2453.653	2601.790	2746.891	2912.736	3088.588	3275.064	3472.791	3682.463	3904.787
8	4122.556											

*Capitals indicate white keys, small type indicates black keys, e.g. a = alis, b = belis, etc.

(*Continued*)

Table 36 (cont.)

Cents deviation from equal temperament. No further stretch applied

	C	delis	D	elis	E	F	gelis	G	alis	A	belis	B
0	A=432.00									−11.2	−9.7	−8.2
1	−14.3	−12.8	−11.3	−9.8	−8.3	−14.3	−12.9	−11.4	−9.9	−8.4	−6.9	−5.4
2	−11.5	−10.0	−8.5	−7.0	−5.5	−11.5	−10.1	−8.6	−7.1	−5.6	−4.1	−2.6
3	−8.7	−7.2	−5.7	−4.2	−2.7	−8.8	−7.3	−5.8	−4.3	−2.8	−1.3	0.2
4	−5.9	−4.4	−2.9	−1.4	0.1	−6.0	−4.5	−3.0	−1.5	0.0	1.5	3.0
5	−3.1	−1.6	−0.1	1.4	2.9	−3.2	−1.7	−0.2	1.3	2.8	4.3	5.8
6	−0.3	1.2	2.7	4.2	5.7	−0.4	1.1	2.6	4.1	5.6	7.1	8.6
7	2.5	4.0	5.5	7.0	8.5	2.4	3.9	5.4	6.9	8.4	9.9	11.4
8	5.3											

Cents deviation from equal temperament with an additional 'small grand' stretch applied

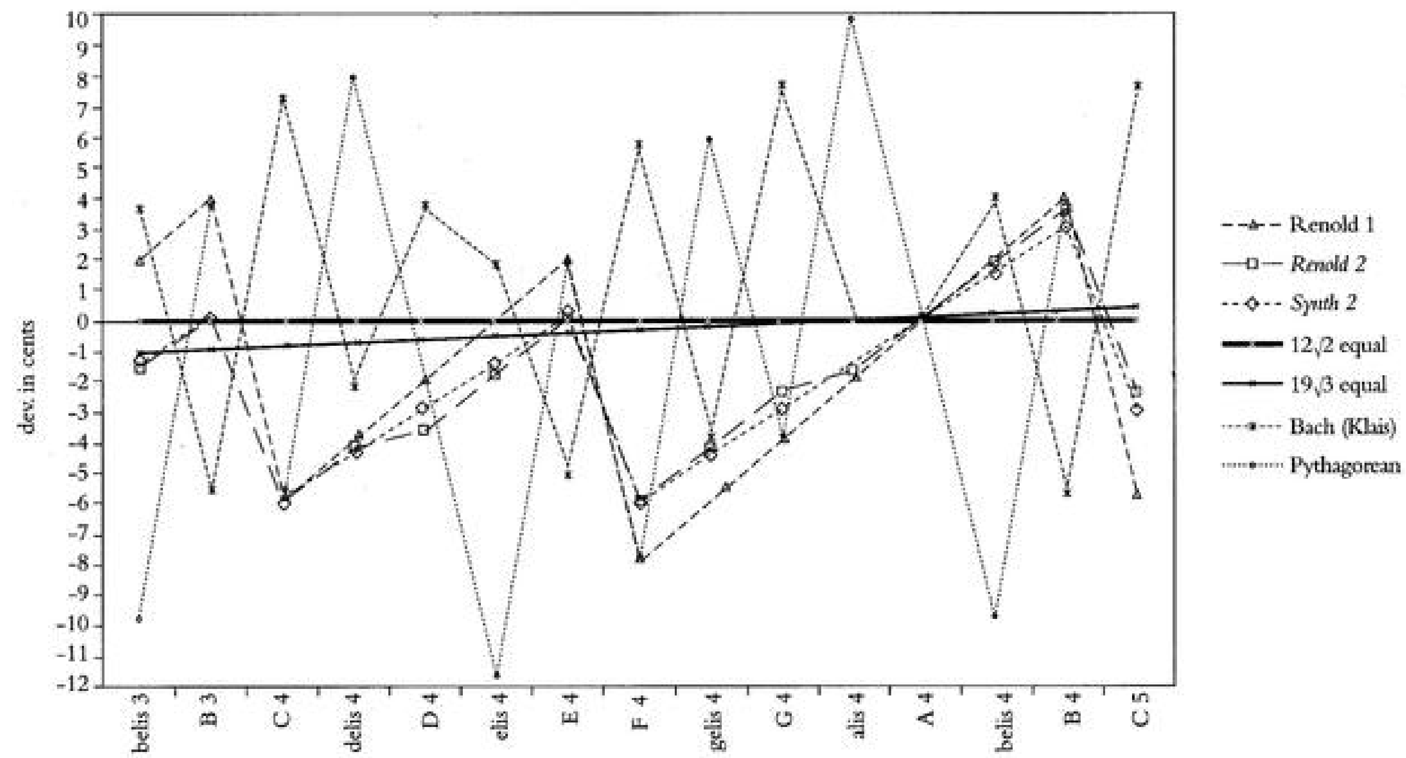
	C	delis	D	elis	E	F	gelis	G	alis	A	belis	B
0	A=432.00									−23.2	−21.1	−19.1
1	−24.6	−22.6	−20.6	−18.6	−16.6	−22.4	−20.3	−18.4	−16.5	−14.6	−12.7	−10.8
2	−16.5	−14.6	−12.7	−10.9	−9.1	−14.7	−13.0	−11.1	−9.2	−7.4	−5.6	−3.8
3	−9.7	−8.0	−6.3	−4.6	−2.9	−8.9	−7.3	−5.8	−4.3	−2.8	−1.3	0.2
4	−5.9	−4.4	−2.9	−1.4	0.1	−6.0	−4.5	−3.0	−1.5	0.0	1.5	3.0
5	−3.1	−1.6	−0.1	1.4	3.0	−3.0	−1.3	0.4	2.1	3.9	5.8	7.8
6	2.2	4.2	6.3	8.4	10.5	5.0	7.1	9.2	11.4	13.5	15.6	17.6
7	12.1	14.2	16.3	18.4	20.5	15.0	17.1	19.2	21.3	23.4	25.5	27.6
8	22.0											

Graphs 1–4 show that my results (referred to as 'Synth R2') closely resemble Maria Renold's later tuning (referred to as Renold 2) and the degree to which intervals of some other tunings differ (including her pre-1991 version—referred to in the graphs as Renold 1). I have used the 12th root of 2 equal temperament as a reference line and have added some just (beatless) lines as well.

With the confidence of a decent synthesis within the temperament octave I calculated frequencies for the entire piano range for the purpose of further study. I should point out that if one were to merely calculate any temperament and copy it onto a piano that the results will likely be poor to miserable. This tuning in particular must be done with the ear as the major thirds, sixths and tenths are very finicky and unforgiving. I must test these intervals constantly while tuning (not tune them directly, but test). I find it best to test them by playing brief passages of music employing them rather than just banging them, out of musical context. I used this table to pre-tune a piano formerly tuned to A = 440. The ear tires easily and I'd rather not waste careful listening on merely the lowering of pitch.

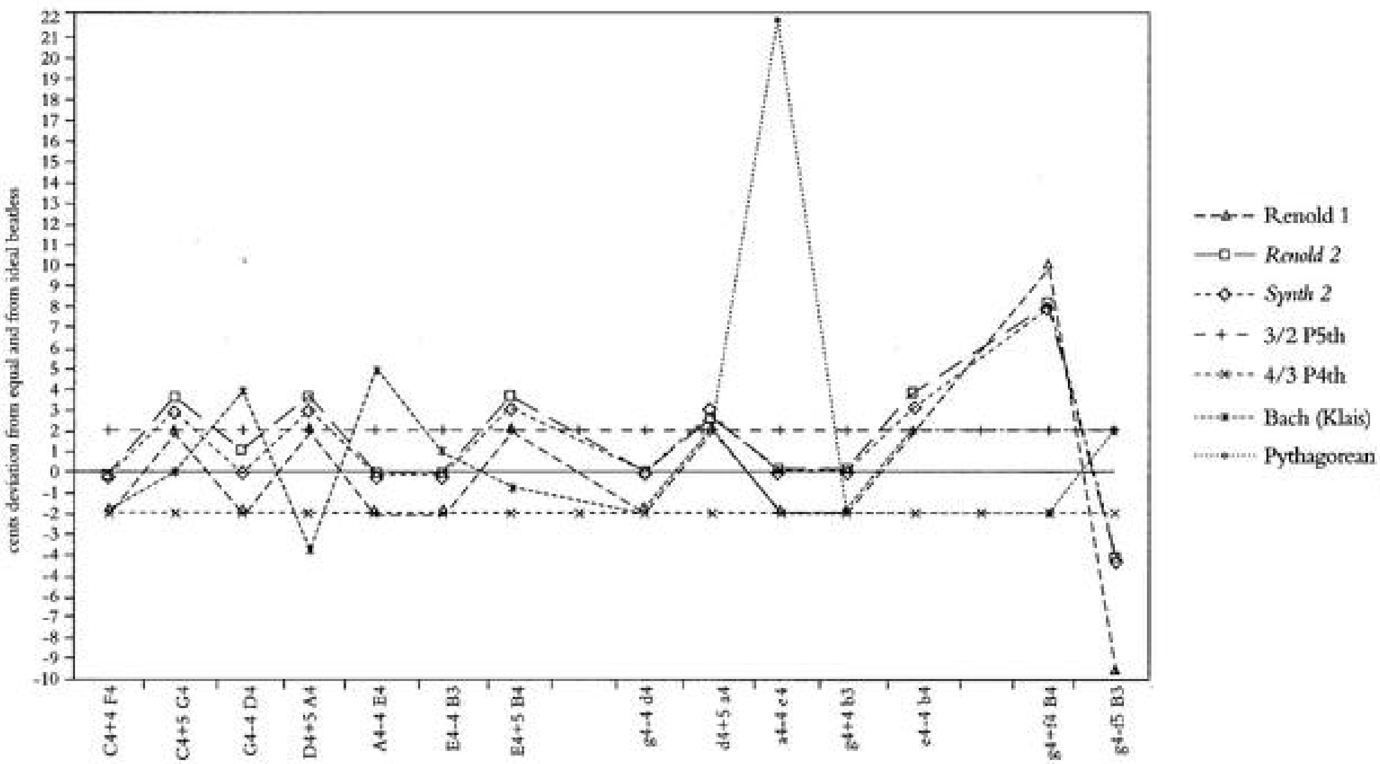
Graph 1

Comparison of temperaments in chromatic scale



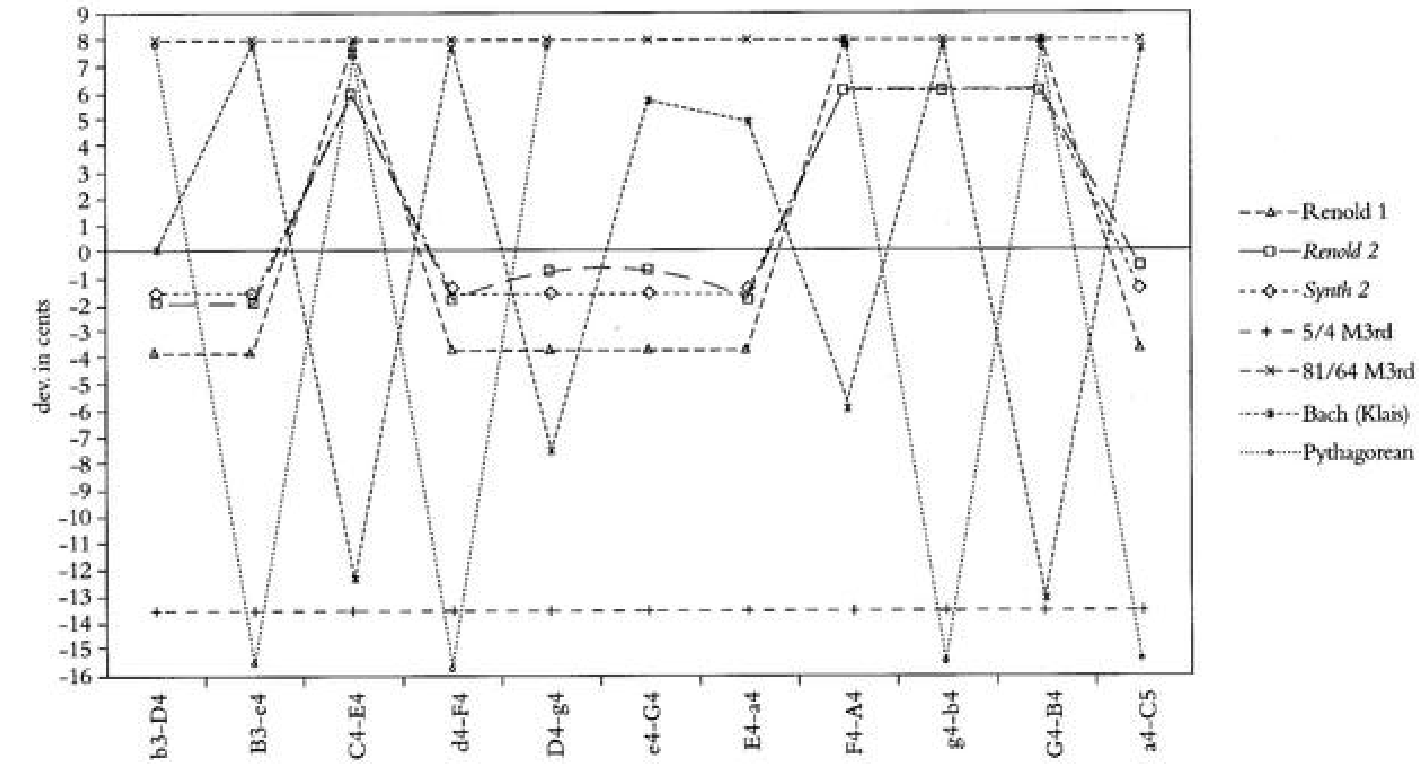
Graph 2

Comparison of fourths and fifths*

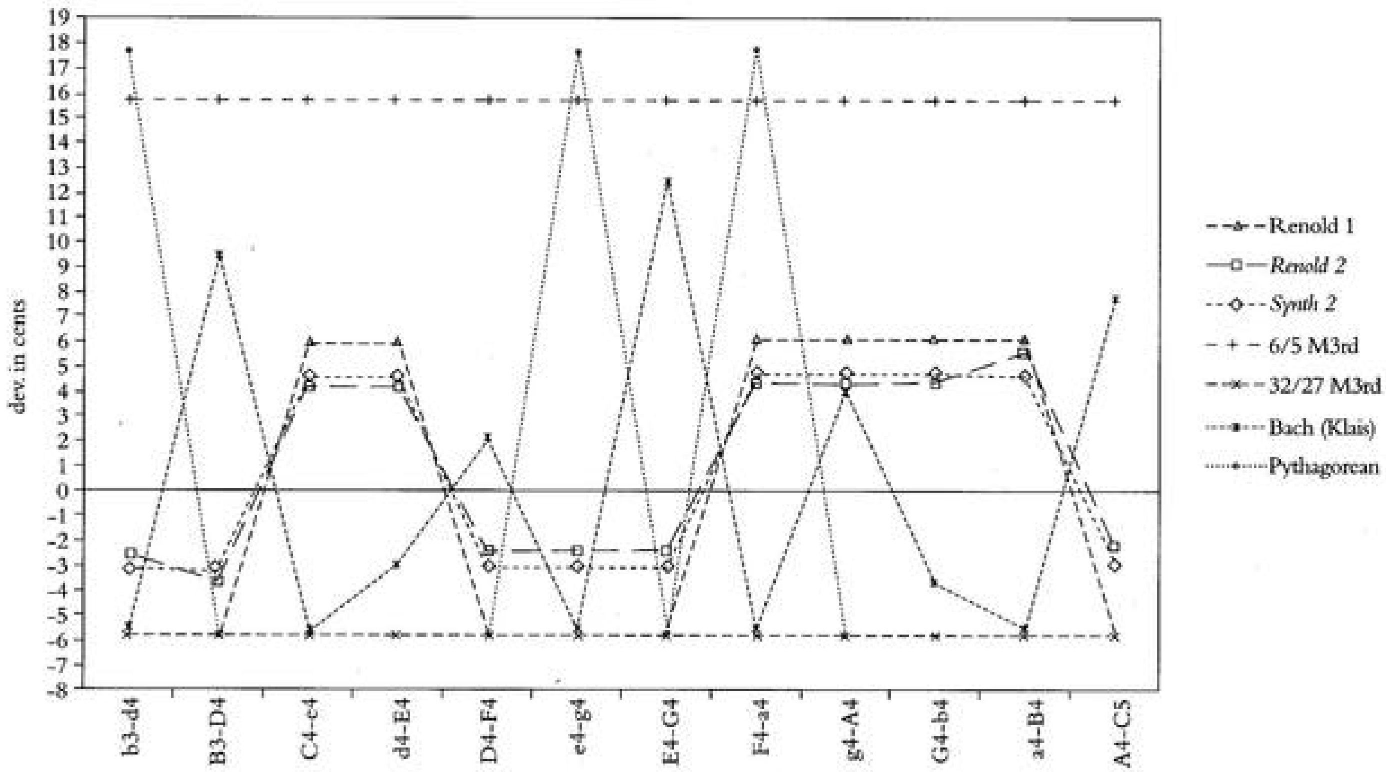


* +4 and +5 indicate up a fourth or fifth; –4 and –5 indicate down a fourth or fifth.

Graph 3
Comparison of major thirds



Graph 4
Comparison of minor thirds



Appendix 3 Addresses of Instrument Makers Known to Us Who Build Instruments Tuned to c = 128 Hz and a^1 = 432 Hz

Oboes, Clarinets, Bassoons

Guntram Wolf
Modern and Historical Wooden Wind Instruments
—Building, Restoration, Repair—
Im Ziegelwinkel 13
D-96317 Kronach
Tel. 0049 9261/4207; Fax 0049 9261/52782

The firm offers all instruments in a variety of tunings. It is also possible to provide instruments with a second tenor joint (*Oberstueck*) or wing (*Fluegel*) in another tuning so that one only needs to change these pieces in order to change from a = 440 Hz to a = 432 Hz, with a negligible change in tone colour and perfect intonation throughout. The instrument is nevertheless favourably priced.

Flutes

Firma Mollenhauer
Postfach 709
D-36041Fulda
Tel. 0049 661 42051
www.mollenhauer.com

French Horn, Trumpet, Trombone, Tuba

Approach the instrument builder in your area with the indications given by Guenter Blechert on p. 162 of this book.

Harp

It is also possible for any instrument maker to make the larger disks (see p. 162).

Organs

George Schamberger
Organ and Musical Instrument Builder
Freie Strasse 33
CH-8610 Uster
Tel. 0041 1 9402935

Schwendener Organ Builder Bodensee
P. Kraul and G. Joly
Schwende 5
D-88643 Herdwangen
Tel. 0049 7557 8698

Chimes and Bars

Taomer Ebersold
Kunstgewerbeatelier

CH–8135 Langnau/Zurich
Tel. 0049 1 7132141

Recorders

Firma Mollenhauer
Joachim Kunath
Postfach 709
D-36041 Fulda
Tel. 0049 661 42051

The firm provides recorders in pentatonic, diatonic and chromatic tuning as well as in twelve fifth-tones tuning.

Chimes

Glockenhausvertrieb
Buergenstegener Weg 5
D-74582 Gerabronn
Tel. 0049 7952 1214

Chimes in pentatonic, diatonic and chromatic tuning as well as in the twelve fifth-tones tuning are available.

Stringed Instruments

Harthmuth Weidler
Geigenbaumeister
Pirckheimerstr. 92
D-90409 Nuernberg
Tel. 0049 911 552721

Contact address and information:
Dietrich Marx
Zur Hege 21
D-35041 Marburg
Tel. 0049 6427 931780

Artur J. C. Bay
Geigenbaumeister, Streichinstrumentenbau
Schweizerhaus
D-88633 Heiligenberg
Tel. 0049 7544 98084

Monochords

Michael Nye
Sunnyside
Stockend, Edge, GL6 6PL
Great Britain
Tel. 0044 1452 814372

Lyres

John Bryan Woodcraft Studio
Unit 12, Salmon Springs Estate
Painswick Road
Stroud GL6 6NU
Great Britain
Tel. 0044 1453 764100

Pianos

Carl A. Pfeiffer
Fluegel- und Klavierfabrik
Neue Ramtelstr. 48
D-71229 Leonberg
Tel. 0049 7152 9760–00

Tuning Forks

Rudolf Wittner & Co. GmbH
Buehlbergstr. 5–6
D-88316 Isny
Tel. 0049 7562 7040

Glossary

Ahrimanic	Having the quality of excessive contraction. Hardening.
Alis, belis, etc.	Tones that are the geometric mean between, for example, A flat and G sharp. In other words, they are geometric mean tones.
Altered tones	Tones chromatically raised or lowered by a minor second = sharpened or flattened.
Apotomé	A variety of semitone. See Table 22.
Augmented	Major or perfect intervals that are enlarged by a semitone.
Aulos	Greek reed instrument
Cents	The specification 'cents' for the intervals is indispensable. We are indebted to the English physicist Alexander Ellis[14] for the introduction of this term. He divided the equal-tempered minor second into 100 and the octave into 1200 equally large parts which he called cents. The size of every interval can thus be given as a simple number, which is easier to oversee and more exact then the normal interval names.[21,22,24,25]
Comma	A variety of semitone. See Table 22.
Degrees of the scale	First tone, I, tonic; second tone, II, supertonic; third tone, III, mediant; fourth tone, IV, subdominant; fifth tone, V, dominant; sixth tone, VI, submediant; seventh tone, VII, leading note.
Didymus comma	A variety of semitone. See Table 22.
Difference tone	A tone which sounds at a pitch (Hz) being the difference between the pitches (Hz) of two other tones.
Diminished	Minor or perfect intervals which are made smaller by a semitone.
Equal-tempered	See chapter 5.
Eurythmy	The art of movement to speech and music as inaugurated by the founder of Anthroposophy, Rudolf Steiner.
Fau	The tone just F as found in the aulos modes.
Formed intervals	The interval between an unaltered tone and a geometric mean tone in the scale of twelve fifths.
Generic tone, Arche	The tone from which an aulos mode arises.
Geo	Modal G.
Hz	The frequency of a tone is measured in Hertz = Hz after the physicist Heinrich Hertz; in English-speaking countries, the term vibrations or cycles per second = c.p.s. is also used.
Interval	The space between two tones. Intervals are generally divided into three groups: perfect, usually considered to have only one consonant size, e.g. fourths, fifths and octaves; imperfect, having two sizes, one smaller called minor, one bigger called major, e.g. thirds and sixths; dissonant, e.g. seconds, sevenths and augmented or diminished intervals.
Just	See chapter 2.
Limma	See Table 22.
Luciferic	Having the quality of excessive expansion and warmth.
Mese	A lower octave of the generic tone within the range of a mode.
Modal	See chapter 2.
Monochord	An ancient apparatus consisting of a resonance box with originally one and later several strings.

Octave displacement	The shifting of a tone through lowering or making it higher by an octave. Mathematically this means doubling or halving the tone's frequency.
Open intervals	Minimally enlarged intervals. See further chapter 22.
Overtones	See chapter 3.
Partials	The tones of the undertone or overtone row.
Pitch of a tone	The pitch of a tone is indicated throughout the book in two ways: 1) by its frequency in Hz; 2) by the range in which it is found. The latter is indicated as follows. The seventh from middle c to b is indicated by small letters, the octaves above with a corresponding numeral in superscript to the right of the letter. The first octave below middle c is given in capitals and the octaves thereafter are indicated by a numeral in subscript to the left of the letter.
Relative key	The scale beginning on the sixth degree of a scale is called the relative key because it has the same key signature, ie, the same number of sharps or flats in the scale.
Scale of twelve fifths	See chapter 2.
True	See chapter 2.
Twelfth tone row	A row of tones found in the overtone and undertone rows of a tone, each tone a twelfth (octave and a fifth) apart. This row forms the true tones. (See also chapter 4.)
Twelfth tones	Tones belonging to the twelfth tone row. Also called fifth tones but here usually referred to as true tones.
Undertones	See chapter 3.

Bibliographic References

1 Baltz, K., *Rudolf Steiners musikalische Impulse. Zweite erweiterte Auflage Goetheanum-Buecher.* Dornach: Philosophisch-Anthroposophischer Verlag am Goetheanum 1981.

2 Walter, B., *Von der Music und vom Musizieren.* S. Fischer Verlag 1957.

3 Steffen, A., 'Musik', in: *Der Kuenstler und die Erfuellung der Mysterien.* Dornach: Verlag fuer Schoene Wissenschaften 1928.

4 Nicomachos von Gerasa. *Manuale Harmonicum.* Tr. C. E. Ruelle. Paris: Baur Libraire Editeur 1881.

5 Boethius, *Fuenf Buecher ueber Musik.* Tr. O. Paul. Leipzig: Verlag F. E. C. Leuckart 1872.

6 Steiner, R., *The Etherization of the Blood.* In GA 130. Tr. A. Freemann, D. S. Osmond. London: Rudolf Steiner Press 1971.

7 Pfrogner, H., *Lebendige Tonwelt.* Muenchen: Langen Mueller Verlag 1976.

8 Schlesinger, K., *The Greek Aulos.* London: Methuen 1939.

9 Hamilton, E., *The Modes of Ancient Greece.* Privately duplicated for colleagues.

10 White, H. E., *Classical and Modern Physics.* New York: D. van Nostrand 1940.

11 Bindel, E., *Die Zahlengrundlagen der Musik in Wandel der Zeiten.* Stuttgart: Verlag Freies Geistesleben 1950–53.

a Band I.

b Band II.

c Band III.

12 Renold, M., 'Die 12 reinen Quintentoene der Chromatischen Tonleiter', in *Das Goetheanum,* No. 5, 1970.

13 Helmholtz, H., *On the Sensation of Sound.* Tr. A. Ellis. New York: Dover Edition, Dover Publications 1954.

14 Ellis, A., Appendix XX and notes to Helmholtz.

15 Herman, C., *Intonation auf Streichinstrumenten. Melodisches und harmonisches Hoeren.* Basel: Baerenreiter Verlag 1964.

16 Stanford, C.V., and Forsyth, C. A., *History of Music.* New York: Macmillan 1930.

17 von Lange, A., *Man, Music and Cosmos.* Tr. F. Hough, revised by N. Hummel. London: Rudolf Steiner Press.

18 Steiner, R., *Die Entstehung und Entwickelung der Eurythmie.* GA 277a. Dornach: Rudolf Steiner Verlag 1965.

19 Riemann, H., *Musik Lexikon,* 9. Auflage. Berlin: Verlag Max Hesses 1919.

20 Steiner, R., 'Okkulte Zeichen und Symbole'. Koeln, 28 and 29 December 1907, in *Nachrichtenblatt der Wochenschrift Das Goetheanum,* Nr. 32–9, 1948, S. 130.

21 De Prony, R., *Instruction élementaire sur les moyens de calculer les intervalles musicaux.* Paris 1832.

22 Bellermann, J. G. H., *Die Groesse der Musikalischen Intervalle als Grundlage der Harmonia.* Berlin 1873.

23 Ellis, A. J., 'Tonometrical Observations on Some Existing Non-harmonic Scales', in *Proceedings of the Royal Society,* London 1884.

24 Reinecke, H.-P., 'Cents Frequenz Periode', in *Untersuchungstabelle fuer musikalische Akustik und Musikethnologie.* Berlin: Walter de Gruyter und Co. 1907.

25 Ruland, H., *Expanding Tonal Awareness.* London: Rudolf Steiner Press 1992.

26 Steiner, R., *The Gospel of St John and its Relation to the Other Gospels* (GA 112), especially lecture given in Kassel on 28 June 1909. Tr. S. & L. Lockwood. New York: Anthroposophic Press 1982.

27 Steiner, R., *Secrets of the Threshold* (GA 147). Tr. nn. London & New York: Anthroposophical Publishing Co. & Anthroposophic Press 1928.

28 Steiner, R., *Die Polaritaet von Dauer und Entwickelung im Menschenleben. Die Kosmische Vorgeschichte der Menschheit* (GA 184). Dornach: Rudolf Steiner Verlag 1959. Lectures of 25, 26 and 28 August 1913 (Muenchen).

29 Steiner, R., *Mystery Knowledge and Mystery Centres* (GA 232). Tr. E. H. Goddard & D. S. Osmond. London: Rudolf Steiner Press 1997.

30 Steiner, R., *The Inner Nature of Music and the Experience of Tone* (GA 283). Tr. M. St Goar. Lectures of 7 and 8 March 1923. New York: Anthroposophic Press 1983.

31 Steiner, R., *Occult Science* (GA 13). Tr. G. & M. Adams. Rudolf Steiner Press, England 1962.

32 Steiner, R., *The Spiritual Hierarchies and their Reflection in the Physical World* (GA 110). Tr. R. M. Querido. New York: Anthroposophic Press 1983.

33 Steiner, R., *Eurythmy as Visible Singing* (GA 278). Tr. A. Stott. Stourbridge: Robinswood Press 1996.

34 Yasser, J., *A Theory of Evolving Tonality*. New York: American Library of Musicology, Contemporary Series, Vol. 1, 1932.

35 Kiesewetter, R. G., *Die Musik der Araber*. Leipzig: Verlag Breitkopf und Haertel 1842.

36 *Encyclopedia Americana*, 1948 Edition, Vol. 19, pp. 283–5.

37 Plato, *Timaeus*. Loeb Classical Library, Vol. VII. R.G. Bury. London: Harvard University Press 1961.

38 Sachs, K., *The Rise of Music in the Ancient World*, East and West. New York: W. N. Norton and Co. 1943.

39 Barbour, J. M., *Tuning and Temperament. A Historical Survey*. Michigan State College Press 1953.

40 Schreyber, H., (Grammateus, Henricus), *Ayn New Kuenstlich Buech*. Nuernberg 1518.

41 Blume, W., *Musikalische Betrachtungen im geisteswissenschaftlichen Sinn*. 2. Auflage. Dornach: Philosophisch-Anthroposphischer Verlag 1985.

42 Steiner, R., *Colour* (GA 291). Tr. J. Salter, P. Wehrle. London: Rudolf Steiner Press 1992.

43 Steiner, R., *The Evolution of Consciousness* (GA 227). Tr. V. E. Watkin, C. Davy. London: Rudolf Steiner Press 1966.

44 Abraham, O., 'Das Absolute Tonbewusstsein', in *Sammelbaende der Internationalen Musikgesellschaft. Dritter Jahrgang 1901/02*. Leipzig: Breitkopf und Haertel.

45 Steiner, R., *First Scientific Lecture Course* (GA 320). Tr. G. Adams. Clent: Goethean Science Foundation 1949.

46 Haba, A., *Mein Weg zur ¼- und ⅙-Ton-Musik*. Duesseldorf: Verlag der Gesellschaft zur Foerderung der systematischen Musikwissenschaft 1971.

47 Hindemith, P., *Unterweisungen im Tonsatz*. Band 1. Mainz: Verlag Schott Soehne 1939. Tr. M. Adams. London: Anthroposophical Co. 1961.

48 Steiner, R., *Der uebersinnliche Mensch, anthroposophisch erfasst*. (GA 231) 3. Auflage. Dornach: Rudolf Steiner Verlag 1982. Darin Vortrag 3 von 3. November, Nachmittag (den Haag).

49 Steiner, R., *Geistige Zusammenhaenge in der Gestaltung des menschlichen Organismus* (GA 218). 2. Auflage. Dornach: Rudolf Steiner Verlag 1976.

50 van der Pals, L., *The Human Being as Music*. Tr. Alan Stott. Stourbridge: Robinswood Press 1992.

51 Steiner, R., 'Erden- und Menschheitsentwickelung. Fragenbeantwortung', in *Nachrichtenblatt der Rudolf Steiner Nachlassverwaltung*, Nr. 26, Sommer 1969.

52 Doerfler, W., *Das Lebensgefuege der Musik*, Band 1. Dornach: Philosophisch-Anthroposophischer Verlag 1975.

53 Steiner, R., *The Fall of the Spirits of Darkness* (GA 177). Tr. A. R. Meuss. London: Rudolf Steiner Press 1993.

54 Steiner, R., *Anthroposophical Leading Thoughts* (GA 26). Tr. G. Adams. London: Rudolf Steiner Press 1985.

55 Steiner, R., *Soul Economy and Waldorf Education* (GA 303). Tr. R. Everett. New York: Anthroposophic Press 1986.

56 Steiner, R., *Art in the Light of Mystery Wisdom* (GA 271). Lecture of 12 September 1920. Tr. J. Collis. London: Rudolf Steiner Press 1970.

57 Steiner, R., See 26.

58 Steiner, R., *Christ and the Human Soul* (in GA 155). Tr. C. Davy. London: Rudolf Steiner Press 1974.

59 Steiner, R., *True and False Paths in Spiritual Investigation* (GA 243). Tr. A. H. Parker. London: Rudolf Steiner Press 1985.

60 Steiner, R., *Universe, Earth and Man* (GA 105). Tr. revised by M. Cotterel et al. London: Rudolf Steiner Press 1941.

61 Steiner, R., *Egyptian Myths and Mysteries* (GA 106). Tr. N. Macbeth. New York: Anthroposophic Press 1971.

62 Wanzloeben, S., *Das Monochord als Instrument und System*. Halle a.d. Saale: Verlag Max Niemeyer 1911.

63 Schering, A., *Geschichte der Musik in Beispielen*. Leipzig: Breitkopf und Haertel Verlag 1931.

64 Bach, J. S., *Das Wohltemperierte Klavier*. Edition Peters.

65 Zuccoli, E., *Aus der Arbeit der ersten Eurythmie-Schule in Stuttgart*, 'Ueber die Tonspirale', S. 24, *and* 'C ist immer Prim', S. 8. Walter Keller Verlag 1980.

66 Steiner, R., *The Mission of the Archangel Michael* (GA 194). Tr. L. D. Monges. New York: Anthroposophic Press 1961.

67 Steiner, R., *The Gospel of St Matthew* (GA 123). Tr. D. S. Osmond, M. Kirkaldy. London: Rudolf Steiner Press 1965.

68 Ellis, A. J., Mendel, A., *Studies in the History of Musical Pitch*. Amsterdam: Fritz Knuf 1968.

69 Kisseleff, T., *Eurythmie. Erinnerungen aus den Jahren 1912–1927*. Malsch/Karlsruhe: Waldhaus Verlag 1949.

70 Schubert, I., *Selbsterlebtes im Zusammenarbeiten mit Rudolf Steiner und Marie Steiner*. Basel: Zbinden Verlag 1970.

71 Schoenberg, A., *Harmonielehre*. Wien 1911. *Models for Beginners in Composition*. New York: Verlag G. Schirmer 1947.

72 Steiner, R., *The East in the Light of the West* (GA 113). Tr. nn. London: Rudolf Steiner Publishing Co. 1940.

73 Steiner, R., See 67. Lecture of 9 September 1910.

74 Junghans, H., *Der Piano- und Fluegelbau*. 4. verbesserte und erweiterte Auflage. Frankfurt am Main: Verlag Das Musikinstrument 1971.

75 Tartini, G., *Trattato di musica secondo la vera scienza dell'armonia*. 1754; *De' Principj dell'armonia musicale contenuta nel diatonico genere*. 1767.

76 Sorge, G. A., *Vorgemach der musikalischen Komposition*. 3 Teile, 1745–7.

77 Audsley, G. A., *The Art of Organ Building*. 1905. Republished in 2 volumes, New York: Dover Publications 1965.

78 Ellerhorst, W., *Handbuch der Orgelkunde*. Hilversum: Verlag Frits Knuf 1966.

79 Cantz, R., *Wesenszüge der Elektrizität in Experimenten und typischen amendungun*. Duernau: Verlag der Kooperative 1988.

80 Cantz, R., 'Konzertwiedergabe in der Wohnstube'. *Elemente der Naturwissenschaft*, Nr. 48, Heft 1, 1988.

81 Deighton, H., Palemo, G., Winter, D., *Singing and the Etheric Tone. Gracia Ricardo's Approach to Singing Based on Her Work with Rudolf Steiner*. New York: Anthroposophic Press 1991.

82 Renold, M., 'Die Tonhoehe c = 128 Hz und eine latente neue Gehoersfaehigkeit des Menschen', in *Das Goetheanum. Wochenschrift fuer Anthroposophie*, 29 April 1990.

83 Straub, W., *Tonqualitaet und Tonhoehe*. Archiv fuer die gesamte Psychologie, Bd. 69. Leipzig 1909.

84 Winkelmann, A., *Ein Appunscher Sonomesser*, Handbuch der Physik, 2. Band: Akustik. Leipzig 1909.

85 Ahlers, E., *Der Ton c^1 = 256 Hz als Grundlage fuer eine zeitgemaesse Stimmung*. Abschlussarbeit am Institut fuer Waldorfpaedagogik Annener Berg. Witten-Annen 1989. (Der hier zitierte Text wurde gegenueber dem der Abschlussarbeit leicht veraendert.)

86 Bachem, A., 'Absolute Pitch', in *Journal of the Acoustical Society of America*. Volume 9, October 1937 and Volume 27 November 1955.

87 Patzlaff, R., *Recht auf Kindheit—ein Menschenrecht*. Heft 4: Kindheit verstummt. Herausgegeben von der Internationalen Vereinigung der Waldorfkindergarten E.V. Stuttgart. Partner der 'Alliance for Childhood'. See also *Erziehungskunst Sonderdruck*, 7 August, 1999.

88 Beckh, H., *Vom geistigen Wesen der Tonarten*, now in its third, expanded and revised edition with the title *Die Sprache der Tonart*. Stuttgart: Urachhaus 1997.

89 Gloeckler, G. & M., 'Die Zahl 70 und ein musikalisches Geheimnis des platonischen Weltenjahres', in *Das Goetheanum. Wochenschrift fuer Anthroposophie*, Mitteilungs Blatt, 25 May 1995.

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Table 21
List of scale tonics

	Hz	Just major scale	Interval to C		Relative just minor scale	Hz	
Descending intervals	89.898 91.022	true G flat just G flat	augmented 4th	major sixth	just E flat true E flat	76.800 75.852	Descending intervals
	67.424 68.267	true D flat just D flat	major 7th	whole tone	just B flat true B flat	115.200 113.778	
	101.136 102.400	true A flat just A flat	major 3rd	fifth	just F true F	86.400 85.334	
	75.852 76.800	true E flat just E flat	major 6th	**prime**	just C **true C**	129.600 **128.000**	
	113.778 115.200	true B flat just B flat	whole tone	fifth	just G true G	189.630 192.000	Ascending intervals
	85.334 86.400	true F just F	fifth	whole tone	just D true D	142.222 144.000	
	128.000 129.600	**true C** just C	**prime** **= 128 Hz**	major 6th	just A true A	213.333 216.000	
Ascending intervals	192.000 194.400	true G just G	fifth	major 3rd	just E true E	160.000 162.000	
	144.000 145.800	true D just D	whole tone	major 7th	just B true B	240.000 243.000	
	216.000 218.699	true A just A	major 6th	augmented 4th	just F sharp true F sharp	180.000 182.250	
	162.000 164.025	true E just E	major 3rd	augmented 8ve	just C sharp true C sharp	276.792 273.374	
	243.000 246.037	true B just B	major 7th	augmented 5th	just G sharp true G sharp	202.500 205.031	
	182.250 184.528	true F sharp just F sharp	augmented 4th	augmented 2nd	just D sharp true D sharp	151.875 153.773	
	273.374 276.792	true C sharp just C sharp	augmented 8ve	augmented 6th	just A sharp true A sharp	227.812 230.660	

All just major scales have the same internal interval structure as the just major scale beginning on true C, but each scale has its own particular character. The interval structure is therefore not character forming. This is given by the character of the tonic which in turn is influenced by the character of the interval between the tonic and the common prime C.* If we thus compare the just major scales beginning on true A and true E, the former has an effect like Sun-saturated light streaming out, while the latter shines out as though from within. The tone true A has breadth and light† and makes a 27:16 true major sixth with C, an interval that opens out more than the fifth, while true E makes a 81:64 true major third with C, an interval that lives and shines within the human being.[33]

In the parallel minor scales the characteristic quality of the interval between the tonic and C is also clearly apparent. Thus the tonic of just F minor makes a fifth with true C. Rudolf Steiner said that the

* It is hardly possible to find more appropriate descriptions of the intervals that truly reflect the real situation than those given by Rudolf Steiner.[20, 23, 33] We will therefore base ourselves on his descriptions.

† See aural experiments in chapter 16.

Why is it that certain intervals, scales and tones sound genuine and others false? Is the modern person able to experience a qualitative difference in a tone's pitch? If so, what are the implications for modern concert pitch and how instruments of fixed tuning are tuned?

Maria Renold tackles these and many other questions, providing a wealth of scientific data. Her pioneering work is the result of a lifetime's research into Western music's Classical Greek origins, as well as a search for new developments in modern times. She strives to deepen musical understanding through Rudolf Steiner's spiritual-scientific research, and she also elucidates many of Steiner's often puzzling statements about music.

The results of her work include the following discoveries: the octave has two sizes (a 'genuine' sounding octave is bigger than the 'perfect' octave); there are three sizes of 'perfect' fifths; an underlying 'form principle' for all scales can be found; and, most importantly, the discovery of a method of tuning the piano which is more satisfactory than equal temperament. She also gives foundation to some of Rudolf Steiner's statements such as: 'c is always prime' and 'c = 128 Hz = Sun'.

MARIA RENOLD (1917-2003) spent her childhood in the United States, where her parents emigrated to found a eurythmy school in New York. She studied eurythmy and later violin and viola and toured with the Bush Chamber Orchestra and the Bush String Quartet. One of Maria Renold's deeply-felt questions concerned the correct concert pitch. When she heard of Rudolf Steiner's concert pitch suggestion of c = 128 Hz she put it into practice immediately, and experimented with it over many years in America and Europe. She also discovered a new method of tuning the piano, closer to the tuning of stringed instruments, arriving at the concert pitch of a^1 = 432 Hz. First published in German in 1985, her book has become a modern classic of musical research.

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